Book of Abstracts IWOTA 2021

International Workshop in Operator Theory and Applications 2021
Chapman University

July 2021

Contents

1 Introduction **2**

2 Plenary Session Abstracts **4**

3 Semi-plenary Sessions Abstracts **10**

4 Special Sessions Abstracts **16**

4.1 Special Session 1 Applications of Operator Theory in Quantum Physics . . . 16
4.2 Special Session 2 - CANCELLED . . . 21
4.3 Special Session 3 - Hilbert Spaces of Analytic Functions and Applications . . . 22
4.4 Special Session 4 - Infinite Dimensional Analysis and Stochastic Processes . . . 32
4.5 Special Session 5 - Laplacian Eigenfunctions – Counts, Morphologies and Statistics . . . 41
4.6 Special Session 6 - Passive and Dissipative Linear Systems . . . 52
4.7 Special Session 7 - Operator Theory and Interpolation in Several Complex Variables . . . 58
4.8 Special Session 8 - Operators in Hypercomplex Analysis . . . 64
4.9 Special Session 9 - Operator Theory on reproducing kernel Hilbert spaces . . . 74
4.10 Special Session 10 - Pseudo-differential Operators . . . 83
4.11 Special Session 11 - Spectral Theory and Differential Operators 100
4.12 Special Session 12 - Theory of Superoscillations . . . 116
4.13 Special Session 13 - General Session for Contributed Papers . . . 122

1 Introduction

IWOTA History

Operator Theory lies at the intersection of several fields such as analysis, quantum mechanics, theoretical physics, probability, stochastic processes, signal processing, machine learning, and many others. The International Workshop of Operator Theory and Applications (IWOTA) brings together mathematicians working in this variety of fields where applications of operator theory are a common denominator. It is worth noting that 2021 marks the first year when IWOTA will be held on the West Coast of the United States since its inception in *1981 in Santa Monica, CA*.

In 2021 IWOTA will be held twice, once in England and once in the US, as the previously planned IWOTA 2020 (England) had to be postponed due to the global pandemic. The two workshops will have different directions and flavors. The IWOTA in England is more theoretical, geared towards operator algebras. The IWOTA 2021 that we are organizing here, at Chapman University, in the United States is focused on applications in physics, stochastic processes and analysis. We will use the strength of Chapman University in quantum computing, superoscillations, and hypercomplex analysis to have a conference with a flavor quite different not only from the sister conference in England, but also from the IWOTA workshops from the previous years.

The first Workshop in Operator Theory and Applications has been organized in Santa Monica by Prof. J. W. Helton (Chair) in 1981, following the vision of Prof. I. Gohberg. Since then, IWOTA has taken place almost every year. Here are the last few meetings, including locations, and chairpersons: • 2009: Guanajuato, Mexico (N. Vasilevski, Chair) • 2010: Berlin, Germany (C. Trunk, Chair) • 2011: Seville, Spain (A. Montes Rodriguez, Chair) • 2012: Sydney, Australia (T. ter Elst, P. Portal, D. Potapov, Chairs) • 2013: Bangalore, India (T. Bhattacharyya, Chair) • 2014: Amsterdam, Holland (A.C.M. Ran, Chair) • 2015: Tbilisi, Georgia (R. Duduchava, Chair) • 2016: St. Louis, USA (G. Knese, Chair) • 2017: Chemnitz, Germany (A. Bottcher, Chair) • 2018: Shanghai, China (H. Lin, G. Yu, Chair) • 2019: Lisbon, Portugal (A. Bastos, Chair) • 2020 : Lancaster, England (G. Blower, Chair)—postponed to 2021.

Chapman University

The IWOTA conference is held on the Orange Campus of Chapman University, in downtown Orange, California. Chapman University has recently changed to an R2 institution with the goal of becoming an R1 one in the near future. The Mathematics Program at Chapman University is flourishing, as it holds two Endowed Chair positions: the Foster G. and Mary McGaw Professorship in Mathematical Sciences held by Dr. Alpay and the Donald Bren Distinguished Chair in Mathematics, held by Dr. Struppa. The program is also enriched by its two research centers: Center of Excellence in Complex and Hypercomplex Analysis (CECHA, Director: Dr. M. Vajiac), and the Center of Excellence in Computation, Algebra, and Topology (CECAT).

We have a group of mathematicians and physicists that not only have a rich research program, but are also committed to diversifying the student population as well as our faculty. We do have a diverse graduate student population in the Computational and Data Sciences Ph.D. Program at Chapman (acronym CADS), the newly created Brain Institute, as well as the new Fowler School of Engineering. Chapman University is in the process of approving a Mathematics, Physics, and Philosophy Ph.D. Program, in addition to the existing Computational and Data Science Ph.D. Program.

The Physics program also holds the prestigious Institute for Quantum Studies (IQS), started by Dr. Yakir Aharonov, one of the giants of theoretical physics, who continues to be the Institute's director to this day. Dr. Aharonov holds the endowed James J. Farley Professorship in Natural Philosophy and he is a 2009 recipient of the President's National Medal of Science.

2021 IWOTA Scientific Committee:

Daniel Alpay (Chapman), Irene Sabadini (Politechnic Milan), and Daniele Struppa (Chapman)

2021 IWOTA Local Organization Committee:

Daniel Alpay (Chapman), Justin Dressel (Chapman), Polona Durcik (Chapman), Ahmed Sebbar (Chapman), Mihaela Vajiac (Chapman), Cai Waegell (Chapman)

Support

The local organizing committee is thankful for the support of the National Science Foundation and Chapman University, through the following grants, endowed chairs, as well as other Chapman institutions:

- National Science Foundation Grant DMS-2055270:
  PI: M. Vajiac, Co-PI: D. Alpay
- Chapman University On Campus Conference Award:
  PIs: D. Alpay, J. Dressel, M. Vajiac
- Foster G. and Mary McGaw Professorship in Mathematical Sciences
  held by Dr. Alpay
- Donald Bren Distinguished Chair in Mathematics
  held by Dr. Struppa
- Schmid College of Science and Technology
- Center of Excellence in Complex and Hypercomplex Analysis
- Institute for Quantum Studies

2 Plenary Session Abstracts

• Differential operators with singular potentials

Jussi Behrndt

AFFILIATION / INSTITUTION: TU Graz

EMAIL: behrndt@tugraz.at

In this talk we discuss qualitative spectral properties of self-adjoint differential operators. We first briefly review some classical results for Schrödinger and Dirac operators with regular potentials and turn to more recent developments afterwards. Our main objective in this lecture is to investigate singular potentials supported on curves or hypersurfaces, which serve as approximative models for localized short-range potentials. In the case of Dirac operators it is necessary to distinguish certain non-critical and critical cases for the strength of the singular perturbation. In particular, it turns out that Dirac operators with singular potentials in the critical case have some unexpected spectral properties.

• Hyperholomorphic spectral theories and applications

Fabrizio Colombo

Politecnico di Milano

fabrizio.colombo@polimi.it

The aim this talk is to give an overview of the spectral theories associated with the notions of holomorphicity in dimension greater than one. A first natural extension is the theory of several complex variables whose Cauchy formula is used to define the holomorphic functional calculus for n-tuples of operators $(A_1, ..., A_n)$. A second way is to consider hyperholomorphic functions of quaternionic or paravector variables. In this case, by the Fueter-Sce-Qian mapping theorem, we have two different notions of hyperholomorphic functions that are called slice hyperholomorphic functions and monogenic functions. Slice hyperholomorphic functions generate the spectral theory based on the S-spectrum while monogenic functions induce the spectral theory based on the monogenic spectrum. There is also an interesting relation between the two hyperholomorphic spectral theories via the F-functional calculus. The two hyperholomorphic spectral theories have different and complementary applications. We finally discuss

how to define the fractional Fourier's law for nonhomogeneous materials using the spectral theory on the S-spectrum.

References

[1] D. Alpay, F. Colombo, I. Sabadini, *Slice Hyperholomorphic Schur analysis,* Operator Theory: Advances and Applications, 256. Birkhäuser/Springer, Cham, 2016. xii+362.

[2] D. Alpay, F. Colombo, I. Sabadini, *Quaternionic de Branges spaces and characteristic operator function*, SpringerBriefs in Mathematics, Springer, Cham, 2020.

[3] F. Colombo, J. Gantner, *Quaternionic closed operators, fractional powers and fractional diffusion process,* Operator Theory: Advances and Applications, 274. Birkhäuser/Springer, Cham, 2019, VIII, vi+322 pp.

[4] F. Colombo, J. Gantner, D. P. Kimsey, *Spectral Theory on the S-spectrum for quaternionic operators,* Operator Theory: Advances and Applications, 270. Birkhäuser/Springer, Cham, 2018. ix+356 pp.

[5] F. Colombo, I. Sabadini, D. C. Struppa, *Noncommutative Functional Calculus. Theory and Applications of Slice Hyperholomorphic Functions*, Progress in Mathematics, Vol. 289, Birkhäuser, 2011, VI, 222 pp.

[6] F. Colombo, I. Sabadini, D.C. Struppa, *Michele Sce's Works in Hypercomplex Analysis. A Translation with Commentaries*, Birkhäuser/Springer, Cham, [2020], 122 pp.

[7] B. Jefferies, *Spectral properties of noncommuting operators*, Lecture Notes in Mathematics, 1843, Springer-Verlag, Berlin, 2004.

[8] T. Qian, P. Li, *Singular integrals and Fourier theory on Lipschitz boundaries*, Science Press Beijing, Beijing; Springer, Singapore, 2019. xv+306 pp.

• Principles of Energy Harvesting in Stochastic Thermodynamic Engines

Tryphon Georgiou

AFFILIATION / INSTITUTION: University of California, Irvine

EMAIL: tryphon@uci.edu

The recent confluence of three subjects, Stochastic Control, Optimal Mass Transport, and Stochastic Thermodynamics, has allowed deeper understanding of the mechanism by which physical contraptions (whether engineered or biological) can transform heat differentials or, as in the age-long conundrum of Maxwell's demon, information into useful work. Our goal in the talk is to overview some of these developments and highlight the geometric framework that allows quantitive assessments on the performance that stochastic thermodynamic engines are capable of. We will then specifically focus on Brownian gyrating engines that consist of overdamped particles that are fed by sources of stochastic excitation and reside in a controlled potential.

The talk is based on joint works with Rui Fu (UCI), Olga Movilla (UCI), Amir Taghvaei (UCI) and Yongxin Chen (GaTech). Research funding by NSF and AFOSR is gratefully acknowledged.

- ## Time-and-band limiting: finding differential operators that commute with naturally appearing integral operators

F. Alberto Grünbaum

AFFILIATION / INSTITUTION: University of California, Berkeley

EMAIL: albertogrunbaum@yahoo.com

A series of remarkable papers by D. Slepian, H. Landau and H. Pollak (Bell Labs around 1960) show that the operators of time-band limiting admit commuting differential operators. A search for the reason behind this unexpected and extremely useful accident has produced over the year a large collection of new such examples. This has deep connections with nonlinear integrable systems such as the Kadomtsev-Petviashvili equations and its master symmetries and many other areas of mathematics. Natural areas of applications include several areas of signal processing as well as random matrix theory.

- ## Semigroups arising in third order in time dynamics with applications to nonlinear acoustics

Irena Lasiecka

AFFILIATION / INSTITUTION: University of Memphis

EMAIL: lasiecka@memphis.edu

A third-order (in time) Partial Differential Equation (PDE) systems arise naturally in a variety of second order PDE models where time relax- ation parameter accounts for an extra derivative, which then leads to a singularly perturbed dynamics. It has been known since the sixties that such models, even in linear case, may be ill-posed in the sense of semigroups. This has motivated an extensive studies of third order dynamics from the point of view of semi- group theory. A class of third order models arising in nonlinear acoustics will be discussed. Such nonlinear (quasi-linear) Partial Differential Equation (PDE) describes nonlinear propagations of high frequency acoustic waves and it is mo- tivated by an array of applications in engineering and medical sciences-including high intensity focused ultrasound [HIFU] technologies. The important feature is that the model resolves the infinite speed of propagation paradox associated with a classical second order in time equation. Replacing a classical heat transfer by heat waves gives rise to the third order in time derivative scaled by a small parameter $\tau > 0$, the latter represents the thermal relaxation time parameter and is intrinsic to the properties of the medium where the dynamics occurs.

The aim of the lecture is to provide a brief overview of recent results in the area which are pertinent to generation of both linear and nonlinear semigroups and their asymptotic behavior with vanishing relaxation parameter $\tau \geq 0$.

Peculiar features associated with the third order dynamics lead to novel phenomenological behaviors.

• An Overview of the Mathematics of Superoscillations

Daniele Struppa

AFFILIATION / INSTITUTION: Chapman University

EMAIL: struppa@chapman.edu

Yakir Aharonov first identified an apparently odd phenomenon that he called superoscillations in the context of his theory of weak values. Roughly speaking a superoscillatory sequence (or a superoscillation in brief) is a band-limited function that can oscillate faster than the highest frequency that it contains. In this talk, based on a series of papers over the last ten years or so, I will introduce the notion of superoscillations, I will explain the nature of their surprising behavior, and I will discuss in some detail one of the most important question regarding such functions. Specifically, suppose we are considering an initial value problem for the Schrodinger equation, and suppose the initial value is a superoscillating function. The question is whether the solution to this initial value problem is still superoscillating. We usually refer to this question as the longevity question for superoscillations. In the talk I will show in detail the answer for the simple case of the free particle, and I will highlight the general principles of the theory that is necessary to study more complex cases of longevity. I will conclude the presentation with a fairly large literature review and ideas for further explorations.

• Spectral Inclusions for Operators with Spectral Gaps

Christiane Tretter

AFFILIATION / INSTITUTION: University of Bern

EMAIL: tretter@math.unibe.ch

Analytical information about the spectra and resolvents of non-selfadjoint operators is of great importance for applications and numerical analysis. However, even for perturbations of selfadjoint operators there are only a few classical results. In this talk relatively bounded, not necessarily symmetric perturbations of selfadjoint operators with spectral gaps are considered. We present new spectral inclusion results and various modifications e.g. for gaps of the essential spectrum or for infinitely many gaps, and several applications.

• Boundary Feedback Stabilization of Fluids in Besov Spaces of Low Regularity by Means of Finite Dimensional Controllers: 3D Navier-Stokes Equations and Boussinesq Systems

Roberto Triggiani

AFFILIATION / INSTITUTION: University of Memphis, University of Virginia

EMAIL: rtrggani@memphis.edu

We shall present two main recent results. First, the 3D-Navier-Stokes equations can be uniformly stabilized in the vicinity of an unstable equilibrium solution by means of a 'minimally' invasive, localized, boundary-based, tangential, static, feedback control strategy, which moreover is finite dimensional. Finite dimensionality in 3D was an open problem. Its solution required a new, suitable, tight Besov space setting of low regularity. Next, an analogous result for the Boussinesq system, coupling the N-S equations with a heat equation. In both cases, unique continuation properties of suitably over-determined adjoint eigen-problems play a critical role.

• Rigidity for II_1 factors

Stefaan Vaes

AFFILIATION / INSTITUTION: University of Leuven

EMAIL: stefaan.vaes@kuleuven.be

Discrete groups and their actions on probability spaces give rise to II_1 factors. When the group is amenable, by Connes' theorem, we essentially always get the unique hyperfinite II_1 factor. In the nonamenable case, Popa's deformation/rigidity theory has led to striking rigidity theorems, including W^*-superrigidity results where the group and its action can be entirely recovered from the ambient II_1 factor. I will give a survey of some of these results, including the computation of invariants of II_1factors and the challenging problem of deciding whenII_1 factors can be embedded one into the other.

• Invariance of absolutely continuous spectra and quasicentral modulus

Dan-Virgil Voiculescu

AFFILIATION / INSTITUTION: University of California, Berkeley

EMAIL: dvv@math.berkeley.edu

The quasicentral modulus is a numerical invariant for n-tuples of operators which appears to play a key role in normed ideal perturbations of operators and multivariable generalizations of the theorems of Kato-Rosenblum and Weyl-von Neumann-Kuroda. I will discuss recent advances and some open problems

3 Semi-plenary Sessions Abstracts

- **L**inear systems and differential equations in random matrix theory

Gordon Blower, Lancaster University, UK

A linear system (A, B, C) with state space Hilbert space H can be used to define a Hankel integral operator on L^2 which has a Fredholm determinant. An alternative description is in terms of a class of operators studied by Howland, who observed an analogy between Schroedinger differential operators on the real line and the Hankel integral operator with kernel $1/(x + y)$. The Fredholm determinant determines a tau function, which depends upon various parameters in the linear system. As an illustration, the talk give solutions to the sinh-Gordon PDE and Painleve III' transcendental ordinary differential equation. These differential equations arise in random matrix theory, and have applications to MIMO in wireless communications.

The work arises in collaboration with Yang Chen (Macau), and Ian Doust (UNSW, Australia).

- **Q**uaternionic non-self adjoint operators and their spectral theory

Uwe Kaehler, University of Aveiro

One of the principal problems in studying spectral theory for quaternionic or Clifford-algebra-valued operators lies in the fact that due to the non-commutativity many methods from classic spectral theory are not working anymore in this setting. For instance, even in the simplest case of finite rank operators there are different notions of a left and right spectrum. Hereby, the notion of a left spectrum has little practical use while the notion of a right spectrum is based on a nonlinear eigenvalue problem. In the present talk we will recall the notion of S-spectrum as a natural way to consider a spectrum in a noncommutative setting and use it to study quaternionic non-selfadjoint operators. To this end we will discuss quaternionic Volterra operators and triangular representation of quaternionic operators similar to the classic approaches by Gohberg, Krein, Livsic, Brodskii and de Branges. Hereby we introduce spectral integral representations with respect to quaternionic chains and discuss the concept of

P-triangular operators in the quaternionic setting. This will allow us to study the localization of spectra of non-selfadjoint quaternionic operators.

- **S**hift Operators on Harmonic Hilbert Function Spaces on Real Balls and von Neumann Inequality

H. Turgay Kaptanoğlu, Bilkent University, Ankara, Turkey

On harmonic function spaces, we define shift operators using zonal harmonics and partial derivatives, and develop their basic properties. These operators turn out to be multiplications by the coordinate variables followed by projections on harmonic subspaces. This duality gives rise to a new identity for zonal harmonics. We introduce large families of reproducing kernel Hilbert spaces of harmonic functions on the unit ball of $\mathbb{R}^n$ and investigate the action of the shift operators on them. We prove a dilation result for a commuting row contraction which is also what we call harmonic type. As a consequence, we show that the norm of one of our spaces $\breve{\mathcal{G}}$ is maximal among those spaces with contractive norms on harmonic polynomials. We then obtain a von Neumann inequality for harmonic polynomials of a commuting harmonic-type row contraction. This yields the maximality of the operator norm of a harmonic polynomial of the shift on $\breve{\mathcal{G}}$ making this space a natural harmonic counterpart of the Drury-Arveson space.

This is joint work with Daniel Alpay of Chapman University, Orange, CA.

- **C**ertification of quantum devices via operator-algebraic techniques

Laura Mancinska, University of Copenhagen

In this talk I will introduce the concept of self-testing which aims to answer the fundamental question of how do we certify proper functioning of black-box quantum devices. We will see that operator-algebraic techniques can be applied to this area and that there is a close link between self-testing and stability of algebraic relations. We will leverage this link to propose a family of protocols capable of certifying quantum states and measurements of arbitrarily large dimension with just four binary-outcome measurements. One of our main proof ingredients is a certain algebraic analogue of Gowers-Hatami stability theorem for group representations.

This is a joint work with Chris Schafhauser and Jitendra Prakash.

• Bianalytic mappings between free spectrahedra

Scott McCullough

University of Florida

EMAIL: sam@ufl.edu

Many optimization problems in systems and control engineering can be formulated in terms of Linear Matrix Inequalities, LMIs. The solution set of an LMI is a spectrahedron. Polydiscs and Matrix balls are examples of spectrahedra. The fully matricial solution set of an LMI, known synonymously as an LMI domain or free spectrahedron, has close ties to operator systems and related topics such as quantum information theory. The natural class of mappings between free spectrahefra are free analytic maps. This talk will discuss the problem of classifying the free bianalytic maps between a pair of spectrahedra, with some emphasis on automorphisms.

• **O**n a function of G.H. Hardy and J.E. Littlewood

Ahmed Sebbar

In their paper: *Notes on the Theory of Series (XX); On Lambert Series, Proceedings of the London Mathematical Society, Ser. (2), 61 (1936), 257-270*, Hardy and Littlewood considered the series

$$f(z) = \sum_{n=1}^{\infty} \frac{1}{n} \left(1 - e^{-\frac{z}{n}}\right), \quad \Re z > 0. \tag{1}$$

They mention the expansion in terms of a Bessel function:

$$f(z) = 2\log z + 2\gamma - 2\sum_{n=1}^{\infty} \left\{ K\left(\sqrt{2ni\pi z}\right) + K\left(\sqrt{-2ni\pi z}\right) \right\} \tag{2}$$

γ is the Euler's constant. We show that this formula is actually a part of a large construction. We explain the link of (1) to a theorem of Beurling on Riemann zeta function and to the zeta function of some ternary quadratic forms.
This is a joint work with Roger Gay .

• Lipschitzness of operator functions

Anna Skripka

We will discuss Lipschitzness of operator functions with respect to Schatten norms in the case of both compact and noncompact perturbations. The latter naturally arise in problems of mathematical physics and noncommutative geometry. We will consider Lipschitz-type bounds for operator functions and characterizations of operator Lipschitzness in terms of familiar properties of the respective scalar functions. Both the celebrated results for compact perturbations and new results for noncompact perturbations rest on multilinear operator integration, a powerful technical method with a long history in noncommutative analysis.

• Perturbations of periodic Sturm–Liouville operators

C. Trunk, TU Ilmenau, Germany

The work by G.W. Hill in 1886 has led to the 'Hill's equation' for the linear second-order ordinary differential equation with periodic coefficients,

$$\frac{1}{r_0}\left(-\frac{\mathrm{d}}{\mathrm{d}x}p_0\frac{\mathrm{d}}{\mathrm{d}x}+q_0\right)y=\lambda y.$$

The above time-independent Schrödinger equation in one spatial dimension with a periodic potential is used within the description of certain effects of atomic nuclei in a crystal. Here the spectral parameter λ has a physical interpretation as the total energy of an electron, and the band structure of the essential spectrum to regions of admissible and forbidden energies. Moreover, impurities (i.e. perturbations) can lead to additional discrete energy levels in the forbidden regions (i.e. eigenvalues in the gap of the essential spectrum).

Here we investigate the change of the spectrum under L^1-assumptions on the differences of the coefficients. We describe the essential spectrum and the absolutely continuous spectrum of the perturbed operator. If a finite first moment condition holds for the differences of the coefficients, then at most finitely many eigenvalues appear in the spectral gaps. This observation extends a seminal result by the Ukrainian mathematician Rofe-Beketov from the 1960ies.

This is based on joint works with J. Behrndt (Graz), P. Schmitz (Ilmenau), and G. Teschl (Vienna).

• Toeplitz operators on the Bergman space.

NIKOLAI VASILEVSKI

Department of Mathematics, CINVESTAV, Mexico City

EMAIL: nvasilev@math.cinvestav.mx

The talk is intended for a wide audience, not necessarily consisting of experts in the theory of Toeplitz operators, and is a review of the results on the description of algebras generated by Toeplitz operators. We begin with a somewhat surprising and unpredictable result on the existence of a large class of non-isomorphic commutative C^*-algebras generated by Toeplitz operators. As it turned out, their symbols must be invariant under the action of maximal Abelian subgroups of the biholomorphisms of the unit ball.

The next surprise was the discovery of a large number of Banach (not C^*) algebras, which turned out to be, as a rule, not semisimple. The problem here is to find a compact set of maximal ideals and to describe the radical.

Finally we consider non-commutative C^*-algebras generated by Toeplitz operators whose symbols are invariant under the action of a subgroup of some maximal Abelian group of biholomorphisms. It turned out that different types of action of the same subgroup lead to completely different properties of the corresponding algebras.

• Integral representation formulae and residue calculus with applications to interpolation

Alain Yger

University of Bordeaux

yger@math.u-bordeaux.fr

Integral representation formulae with weights (Bochner-Martinelli, Cauchy-Weil, Cauchy-Fantappié, etc.) have been extensively developped since more than 20 years. The same for what concerns multivariate residue theory, which revealed to be quite an efficient tool to provide closed formulae of the Kronecker-Jacobi type solving explicitely the Bézout identity $1 = \sum_{j=1}^{m} q_j p_j$ in the algebraic setting, more generally in weighted algebras of entire functions such that the Paley-Wiener algebra. Despite the fact that such residue calculus highly relies on commutativity, it seems that some technics which support it could be transposed to operator theory (residues being from the beginning defined as traces of operators!). Also the crucial role played by distributions or currents is not so well known outsite the world of multivariate complex analysis. I will present in this talk a selection of examples which motivate the use of such residue theory from a concrete point of view and, at the same time, suggest some transposition to non-commutative horizons. My collaboration with D. Alpay since five years motivated indeed the topics I will discuss in this talk.

References

[1] A. Vidras, A. Yger, Bergman-Weil expansion for holomorphic functions, Math. Annalen (2021), https://doi.org/10.1007/s00208-020-02137-8

[2] M. Sombra, A. Yger, Bounds for multivariate residues and for the polynomials in the elimination theorem, Moscow Math. J. (2021) no. 1, 129-173

[3] M. Andersson, D. Eriksson, H. Samuelsson Kalm, E. Wulcan, A. Yger, Non proper intersection products and generalized cycles, European J. Math. (2021), to appear

[4] A. Yger, Flexibility of analysis serving computational polynomial algebra or Arithmetics, Complex Variables and Operator Theory (2021), 14, no. 3, https://doi.org/10.1007/s11785-021-01080-z

[5] C. A. Berenstein (†), A. Vidras, A. Yger, Multidimensional residue calculus and applications, Monography in finalization (450 pages), Math. Surveys and Monographs (AMS), submission due June 2021

[6] D. Alpay, A. Yger, About a non standard interpolation problem, Computational Methods and Function Theory volume 19 (2019), pp. 97-115

4 Special Sessions Abstracts

4.1 Special Session 1 Applications of Operator Theory in Quantum Physics

- **Some categories of euclidean Jordan Algebras**

Alexander Wilce

Abstract: We exhibit a construction whereby symmetric monoidal categories of special euclidean Jordan algebras can be constructed. One of these, which we call **InvQM**, provides a dagger-compact category containing self-adjoint parts of real, complex and quaternionic matrix algebras. Thinking of the latter as representing irreducible finite-dimensional real, complex and quaternionic quantum mechanical systems, this provides a theory in which systems of all three of these types live together. However, the price one pays for this is that a composite of two quantum systems in **InvQM** is no longer irreducible, but comes with a 2-valued superselection rule. (Joint work with H. Barnum and M. Graydon, Quantum **4** (2020), arXiv:1606.09331)

- **Pointwise convergence of integral kernels for Feynman path integrals**

S. Ivan Trapasso

University of Genoa, Italy

salvatoreivan.trapasso@unige.it

Introduction. The Feynman path integral formulation of quantum mechanics is universally recognized as a milestone of modern theoretical physics. Roughly speaking, the core principle of this picture provides that the integral kernel of the time-evolution operator shall be expressed as a "sum over all possible histories of the system". This phrase entails a sort of integral on the infinite-dimensional space of suitable paths, to be interpreted in some sense as the limit of a finite-dimensional short-time approximation procedure. In spite of the suggestive heuristic insight, the quest for a rigorous theory of Feynman path integrals is far from over, as evidenced by the wide variety of mathematical approaches developed over the last seventy years - cf. [1] and the references therein for a broad introductory account.

Lagrangian formulation via the Trotter formula. Among the several proposed frameworks, the closest one to Feynman's original intuition is probably the time-slicing approximation due to E. Nelson [4]. In short, if $U(t)$ is the Schrödinger time evolution operator with Hamiltonian $H = H_0 + V$ (free particle plus a suitable potential perturbation), then the Trotter product formula holds for all $f \in L^2(\mathbb{R}^d)$:

$$U(t)f = e^{-\frac{i}{\hbar}t(H_0+V)}f = \lim_{n\to\infty} E_n(t)f, \quad E_n(t) = \left(e^{-\frac{i}{\hbar}\frac{t}{n}H_0}e^{-\frac{i}{\hbar}\frac{t}{n}V}\right)^n.$$

Integral representations for the approximate propagators $E_n(t)$ can be derived, so that the Trotter formula allows one to give a precise meaning to path integrals by means of a sequence of integral operators.

The problem of pointwise convergence. Notwithstanding the convergence results in suitable operator topologies, a closer inspection of Feynman's writings suggests that his original intuition underlay the much more difficult and widely open problem of the pointwise convergence of the integral kernels of the approximation operators $E_n(t)$ to that of $U(t)$. In the recent paper [5] we addressed this problem by means of function spaces and techniques arising in the context of time-frequency analysis. The toolkit of Gabor analysis has been fruitfully applied to the study of path integrals only in recent times, leading to promising outcomes [6,7,8].

Main results. With reference to the notation above, we consider a setting where H_0 is the Weyl quantization of a real quadratic form, hence covering fundamental examples such as the free particle or the harmonic oscillator. In addition, we introduce a bounded potential perturbation V whose regularity is characterized in terms of the decay in phase space of its windowed Fourier transform (such levels of regularity are encoded by the so-called modulation spaces). This setting covers, and in fact extends, a case that is often met in the literature on mathematical path integrals - namely, the harmonic oscillator plus a bounded perturbation which is the Fourier transform of a complex (finite) measure (see for instance the pioneering works by K. Itô and the line of research developed by S. Albeverio, R. Høegh-Krohn and S. Mazzucchi).

We exploit techniques of Gabor analysis of pseudodifferential operators to prove that the problem of pointwise convergence has a positive answer under the previous assumptions. Precisely, we prove stronger convergence results which imply uniform convergence on compact subsets for the integral kernels in the Trotter formula.

Our results hold for any fixed value of $t \in \mathbb{R} \setminus \mathfrak{E}$, where $\mathfrak{E}$ is a discrete set of exceptional times - in that case the integral kernels are genuine distributions. In the recent contribution [2] we characterized the properties of such distribution kernels (precisely, they are "mild distributions" in the sense of Feichtinger's Banach-Gelfand fundamental triple of harmonic analysis, cf. e.g. [3]) and we derived weaker convergence results in the sense of modulation spaces even for $t \in \mathfrak{E}$.

References

[1] S. Albeverio and S. Mazzucchi. Path integral: mathematical aspects. Scholarpedia, 6(1):8832 (2011).

[2] H. Feichtinger, F. Nicola and S. I. Trapasso. On exceptional times for pointwise convergence of integral kernels in Feynman-Trotter path integrals. In: Anomalies in Partial Differential Equations, pp. 293-311, Springer-INDAM series (2021).

[3] H. Feichtinger. Classical Fourier analysis via mild distributions. Nonlinear Stud. 26, no. 4, 783–804 (2019).

[4] E. Nelson. Feynman integrals and the Schrödinger equation. J. Math. Phys. 5, 332-343 (1964).

[5] F. Nicola and S. I. Trapasso. On the pointwise convergence of the integral kernels in the Feynman-Trotter formula. Comm. Math. Phys. 376 (2020), no. 3, 2277–2299.

[6] F. Nicola and S. I. Trapasso. Approximation of Feynman path integrals with non-smooth potentials. J. Math. Phys. 60 (2019), 102103.

[7] F. Nicola. On the time slicing approximation of Feynman path integrals for non-smooth potentials. J. Anal. Math. 137(2) (2019), 529–558.

[8] F. Nicola. Convergence in L^p for Feynman path integrals. Adv. Math. 294 (2016), 384–409.

- **Euclidean Green's function for relativistic N particle system**

Shaikh Gohin Samad and Wayne Polyzou
AFFILIATION / INSTITUTION:University of Iowa
EMAIL:shaikhgohin-samad@uiowa.edu

ABSTRACT: In this talk a Euclidean formulation of relativistic quantum mechanics for systems of a finite number of degrees of freedom will be discussed. Relativistic treatments of quantum theory are needed to study hadronic systems at sub-atomic distance scales. Special relativity and quantum mechanics are most naturally combined using relativistic quantum field theory. Quantum field theory is ill defined, but can be used when perturbation theory is justified. Unfortunately the interactions involving quarks are too strong for a perturbative treatment. While direct interaction approaches to relativistic quantum mechanics have proved to be useful, they have two disadvantages. One is that cluster properties are difficult to realize for systems of more than two particles. The second is that the relation to quantum field theories is indirect. Alternative formulations of relativistic quantum mechanics are possible, but it is difficult to formulate theories with all of desired properties. Euclidean formulations of relativistic quantum mechanics motivated by the Euclidean axioms of

quantum field theory(Konrad Osterwalder and Robert Schrader 1974) provide an alternative representation that does not have these difficulties. More surprising, the theory can be formulated entirely in the Euclidean representation without the need for analytic continuation. In this talk Kernels for systems of N free particles of any spin are discussed.Reflection positivity are established for desired kernals. Explicit formulas for generators of the Poincaré group for any spin are constructed and shown to be self-adjoint on the Euclidean representation of the Hilbert space. The structure of correlations that preserve both the Euclidean covariance and reflection positivity are discussed.

This talk is partially based on a joint work with Wayne Polyzou.

References

[1] Gohin Shaikh Samad,Wayne N. Polyzou, *Euclidean formulation of relativistic quantum mechanics of N particles.* Physical Review C

• FUNCTIONAL INEQUALITIES IN NON-COMMUTATIVE $\mathbb{L}_p$ SPACES FOR QUANTUM MANY-BODY SYSTEMS

ANGELA CAPEL

AFFILIATION / INSTITUTION: Technische Universität München

EMAIL: angela.capel@ma.tum.de

The mixing time of quantum Markov semigroups which model dissipative evolutions of open quantum many-body systems can be bounded using optimal constants of certain non-commutative functional inequalities (i.e. functional inequalities in non-commutative $\mathbb{L}_p$ spaces). In our setting, the kernel of the infinitesimal generator of the quantum Markov semigroup has a unique operator, and it is given by the Gibbs state of a local, commuting Hamiltonian (thus, it is, in particular, a quantum Markov chain). We particularly focus here on the non-commutative modified logarithmic Sobolev inequality (MLSI).

For classical (commuting) spin systems, the positivity of MLSI constants follows from a mixing condition for the Gibbs measure, via quasi-factorization results for the entropy. Inspired by the commuting case, in this talk we present a strategy to derive the positivity of the non-commutative MLSI constants associated to the dynamics of certain quantum systems from some decay of correlations on the Gibbs state of local, commuting Hamiltonians. The main ingredient at the core of this strategy are the so-called results of quasi-factorization of the relative entropy.

As an application of this strategy, we obtain the first non-trivial examples of positivity of non-commutative MLSI constants for quantum systems. More specifically, in a series of works ([1]-[5]), we construct a family of inequalities of quasi-factorization of the relative entropy in terms of a conditional relative entropy, which we subsequently employ in several standard dynamics in quantum spin systems, namely the heat-bath and the Davies dynamics, and for different assumptions of clustering of correlations on the Gibbs state, i.e. the unique fixed point of the semigroup. This strategy allows us to solve a long-standing open problem in [5] regarding the existence of a size-independent MLSI constant for quantum Markov semigroups converging to the Gibbs state of a nearest-neighbour Hamiltonian at high enough temperature.

References

[1] A. Capel, A. Lucia and D. Pérez-García, *Superadditivity of quantum relative entropy for general states.* IEEE Transactions on Information Theory

[2] A. Capel, A. Lucia and D. Pérez-García, *Quantum conditional relative entropy and quasi-factorization of the relative entropy.* Journal of Physics A: Mathematical and Theoretical

[3] I. Bardet, A. Capel, A. Lucia, D. Pérez-García and C. Rouzé, *On the modified logarithmic Sobolev inequality for the heat-bath dynamics for 1D systems.* arXiv preprint, arXiv:1908.09004

[4] I. Bardet, A. Capel and C. Rouzé, *Approximate tensorization of the relative entropy for noncommuting conditional expectations.* arXiv preprint, arXiv:2001.07981

[5] A. Capel, C. Rouzé and D. Stilck França, *The modified logarithmic Sobolev inequality for quantum spin systems: classical and commuting nearest neighbour interactions.* arXiv preprint, arXiv:2009.11817

• A Common Parametrization for Finite Mode Gaussian States, their Symmetries and associated Contractions with some Applications

Tiju Cherian John and K R Parthasarathy

Fulbright-Nehru Postdoctoral Fellow / University of South Carolina

EMAIL:tijucherian@gmail.com, tiju@mailbox.sc.edu

ABSTRACT: Let $\Gamma(\mathcal{H})$ be the boson Fock space over a finite dimensional Hilbert space $\mathcal{H}$. We show that every gaussian symmetry admits a Klauder-Bargmann integral representation in terms of coherent states.

Furthermore, gaussian states, gaussian symmetries, and second quantization contractions belong to a weakly closed, selfadjoint semigroup $\mathcal{E}_2(\mathcal{H})$ of bounded operators in $\Gamma(\mathcal{H})$. This yields a common parametrization for these operators. It is shown that the new parametrization for gaussian states is a fruitful alternative to the customary parametrization by position-momentum mean vectors and covariance matrices. This leads to a rich harvest of corollaries:

(i) every gaussian state ρ admits a factorization $\rho = Z_1^\dagger Z_1$, where Z_1 is an element of $\mathcal{E}_2(\mathcal{H})$ and has the form $Z_1 = \sqrt{c}\Gamma(P) \exp \sum_{r=1}^{n} \lambda_r a_r + \sum_{r,s=1}^{n} \alpha_{rs} a_r a_s$ on the dense linear manifold generated by all exponential vectors, where c is a positive scalar, $\Gamma(P)$ is the second quantization of a positive contractive operator P in $\mathcal{H}$, a_r, $1 \leq r \leq n$ are the annihilation operators corresponding to the n different modes in $\Gamma(\mathcal{H})$, $\lambda_r \in \mathbb{C}$ and $[\alpha_{rs}]$ is a symmetric matrix in $M_n(\mathbb{C})$;

(ii) an explicit particle basis expansion of an arbitrary mean zero pure gaussian state vector along with a density matrix formula for a general gaussian state in terms of its $\mathcal{E}_2(\mathcal{H})$-parameters;

(iii) a class of examples of pure n-mode gaussian states which are completely entangled;

(iv) tomography of an unknown gaussian state in $\Gamma(\mathbb{C}^n)$ by the estimation of its $\mathcal{E}_2(\mathbb{C}^n)$-parameters using $O(n^2)$ measurements with a finite number of outcomes.

References

[1] Tiju Cherian John and K. R. Parthasarathy, *A Common Parametrization for Finite Mode Gaussian States, their Symmetries and associated Contractions with some Applications.* Journal of Mathematical Physics 62, 022102 (2021); https://doi.org/10.1063/5.0019413

4.2 Special Session 2 - CANCELLED

4.3 Special Session 3 - Hilbert Spaces of Analytic Functions and Applications

- **Generalized Fock space and fractional derivative**

Natanael Alpay and Paula Cerejeiras and Uwe Kähler

Chapman University (NA) and University of Aveiro (PC and UK)

alpay100@mail.chapman.edu,pceres@ua.pt, ukaehler@ua.pt

The role of Fock spaces - be it in quantum mechanics, stochastic analysis, signal processing or other fields - is of crucial importance as they allow representation of coherent states in terms of position and momentum operators to which the original problem is mapped by appropriated integral transform such as the Bargmann transform.

The classic Fock space can be characterized (up to a positive multiplicative factor) as the only Hilbert space of entire functions in which the adjoint of the derivation is the multiplication by the complex variable, thus its link with quantum mechanics and the Hamiltonian operator. Similarly (and again up to a positive multiplicative factor) the Hardy space can be seen as the only space of functions analytic in the open unit disk for which the adjoint of the backward shift operator is the multiplication operator.

While this is closely linked to classic applications based on the standard derivative in many cases more general concepts of derivates are required. This can take the form of a Dunkl derivative like in Calogero-Sutherland-Moser models or fractional derivatives like in the cased of grey noise stochastic processes based on the Mittag-Leffler function as probability measure. These type of derivatives can be considered as special cases of the Gelfond-Leontiev operator of generalized differentiation.

In this talk we are going to present a general framework of constructing and studying Fock spaces and Hardy spaces with respect to the Gelfond-Leontiev operator of generalized differentiation. In particular we also propose a new characterization of the Hardy space in term of the adjoint of such generalized fractional differentiation operator. We begin by an appropriated definition of such Hardy spaces using reproducing kernel methods. This leads to a Carleman's condition associated to the correspondent Stieltjes moment problem and will allow for a new characterization of the Fock space.

In the end concrete examples like the above mentioned cases will be presented.

• The Douglas-Shapiro-Shields factorization for matrix functions of bounded type

Raúl E. Curto

Department of Mathematics, University of Iowa

Email: raul-curto@uiowa.edu

We consider a factorization question emerging from the Beurling-Lax-Halmos (BLH) Theorem, which characterizes the shift-invariant subspaces of vector-valued Hardy spaces. The BLH Theorem states that a backward shift-invariant subspace is a model space $\mathcal{H}(\Delta) \equiv H^2_E \ominus \Delta H^2_E$, for some inner function Δ. (Here E is a Hilbert space and H^2_E denotes the E–valued Hardy space). Thus, for a subset $F \subseteq H^2_E$, if E_{F^*} denotes the smallest backward shift-invariant subspace containing F, then $E^*_F = \mathcal{H}(\Delta)$, for some Δ inner. On the other hand, if an inner function Δ is given, it is natural to ask about the smallest number of vectors in a subset F satisfying $\mathcal{H}(\Delta) = E_{F^*}$. In search for an answer, we will instead address the more general question of finding a description for the sets F in H^2_E such that $\mathcal{H}(\Delta) = E^*_F$.

The above is intrinsically related to a new canonical decomposition of operator-valued strong L^2-functions (in the sense of V. Peller). Our description includes, as a special case, the Douglas-Shapiro-Shields factorization for matrix functions of bounded type. (Such functions are matrix-valued functions all of whose entries are in the Nevanlinna class, that is, quotients of two bounded analytic functions on the open unit disk.) Specifically, we prove that a strong L^2-function Φ with values in $\mathcal{B}(D, E)$ (D, E Hilbert spaces) can be represented as

$$\Phi = \Delta A^* + B,$$

where Δ is an inner function with values in $\mathcal{B}(E', E)$, Δ and A are right coprime, $\Delta^* B = 0$, and E' is a closed subspace of E.

The talk is based on recent joint work with In Sung Hwang and Woo Young Lee ([1], [2]).

References

[1] R.E. Curto, I.S. Hwang and W.Y. Lee, *Matrix functions of bounded type: An interplay between function theory and operator theory*, Memoirs Amer. Math. Soc. 260(2019), no. 1253.

[2] R.E. Curto, I.S. Hwang and W.Y. Lee, *The Beurling-Lax-Halmos Theorem for infinite multiplicity*, J. Funct. Anal. 280(2021), art. 108884.

• Stability of periodic delay systems and harmonic transfer function

L.BARATCHART, S.FUEYO and J-B.POMET

AFFILIATION / INSTITUTION: INRIA Sophia Antipolis

EMAIL: laurent.baratchart@inria.fr, sebastien.fueyo@inria.fr and Jean-Baptiste.Pomet@inria.fr

This talk deals with linear periodic dynamical systems of the form displayed in equation (3) below, that we call linear periodic difference-delay systems, and focuses on their exponential stability properties:

$$y(t) = \sum_{j=1}^{N} D_j(t) y(t - \tau_j). \tag{3}$$

We consider the (doubly infinite) matrix $H(p)$, depending on a complex variable $p \in \mathbb{C}$ which has the following form:

$$H(p) = I_\infty - \sum_{j=1}^{N} e^{-p\tau_j} L_{D_j} \tilde{D}_{\tau_j}, \tag{4}$$

where L_{D_j} are the (doubly infinite) Laurent matrices associated to the Fourier expansion of $D_j(t)$, $\tilde{D}_{\tau_j}$ is a (doubly infinite) diagonal matrix depending on the delay τ_j and the period of the system, and I_∞ is the identity operator acting on $l^2(\mathbb{Z})$, the space of square-summable sequences. We are able to link the stability properties of System (3) and inversibility properties of the operator $H(\cdot)$ defined in the equation (4):

Theorem 1 *Assume that the D_j are periodic and differentiable with Hölder continuous derivative. Then, a necessary and sufficient condition to have the exponential stability of the origin of System (3) is the existence of a real number $\beta < 0$ such that :*

1. *$H(p)$ is invertible in $l^2(\mathbb{Z})$ to $l^2(\mathbb{Z})$ for all $p \in \{z \in \mathbb{C} | \Re(z) \geq \beta\}$.*
2. *the inverse of $H(\cdot)$, $H(\cdot)^{-1}$, is uniformly bounded in the right half plane $\{z \in \mathbb{C} | \Re(z) \geq \beta\}$.*

Remark 1 *When System* (3) *is time-invariant, we have in this case the Henry-Hale theorem ([**?**, **?**]) which gives a necessary and sufficient condition for the exponential stability of the system. Theorem 1 presented here is an analog of the Henry-Hale theorem in the periodic case. In fact, when the D_j are constant, the theory of the complex almost periodic functions reduces the Theorem 1 to the Henry-Hale theorem. Thus, Theorem 1 is a generalization.*

Remark 2 *Condition 2 is redundant when the delays are commensurable, however we conjecture that it is necessary in general. Still, we have no counter-example as yet.*

Sufficiency follows in a sense the classical lines of Henry-Hale based on Laplace transform, but important additional ingredients are needed in this infinite-dimensional context with the use of a variation-of-constant formula adapted to periodic difference delay system and a controlled inversion in the Wiener algebra adapted to the almost periodic functions with values in a Banach ([**?**, **?**]).

Necessity uses realization theory of control systems and the fact that the operator $H(\cdot)$ can be linked with the monodromy operator *i.e.* the operator solution of System (3) integrated after one period of the system.

In fact electronic engineering guided us to construct the proof of the necessity. System (3) arises in circuit theory. Indeed, when an electric network operates at high frequency, one cannot neglect delays induced by transmission lines connecting the components of the circuit. System (3) represents typically a high frequency limit system of a nonlinear circuit, containing lossless transmission lines, linearized around a periodic trajectory. To check the stability of System (3), textbooks in electrical engineering ([**?**] for example) rely on an input-output system where the input u is a small current which disturbs System (3) and y is the voltage response to this perturbation. The use of Fourier development and the Laplace transform in the input-output system permit to obtain an infinite-dimensional time-invariant system and the operator $H(\cdot)$ introduced in equation (4). The operator $H(\cdot)$ is called *Harmonic Transfer Function* (HTF). Though the HTF is a Hilbert valued analytic map (values: continuous operators $l^2(\mathbb{Z}) \to l^2(\mathbb{Z})$), it is rarely considered from this viewpoint. Moreover, the link between the HTF and the stability of System (3) was not clear from a mathematical perspective that is why this study has been undertaken.

• Teichmüller's theorem in higher dimensions

Anatoly Golberg

Holon Institute of Technology, Holon, Israel

EMAIL:golberga@hit.ac.il

By the Uniformization Theorem, a ring domain $\mathcal{R}$ (a doubly connected domain) of the complex plane $\mathbb{C}$ is conformally equivalent to the annulus $\{z \in \mathbb{C} : r_0 < |z| < r_1\}$ for some $0 \le r_0 < r_1 \le \infty$. The quantity $\log(r_1/r_0)$

is called the modulus of $\mathcal{R}$ and denoted by $\operatorname{mod}\mathcal{R}$. O. Teichmüller [1] showed that a ring $\mathcal{R}$ with $\operatorname{mod}\mathcal{R} > \pi$ separating 0 and ∞ contains a circle centered at 0 and that the constant π is sharp. Indeed, the Teichmüller ring $R_T(t) = \mathbb{C} \setminus \big([-1,0] \cup [t,+\infty)\big)$ with $t = 1$ serves as an extremal case. Teichmüller introduced the Grötzsch ring and the Teichmüller ring and found their extremal properties in [1]. Using the extremal property of the Teichmüller ring, D. A. Herron, X. Liu and D. Minda [2] showed the following sharp result.

Let $\mathcal{R}$ be a ring separating 0 and ∞ in $\mathbb{C}$ with $m = \operatorname{mod}\mathcal{R} > \pi$. Then $\mathcal{R}$ contains an annular subring $\mathcal{A}$ of the form $\{z : r_0 < |z| < r_1\}$ with

$$\operatorname{mod}\mathcal{A} = \log \mu_T^{-1}(m),$$

where $\mu_T(t) = \operatorname{mod} R_T(t)$ for $0 < t < +\infty$. The result is sharp.

From the inequality $\mu_T(t) < \log t + \pi$ for $t > 1$, which is equivalent to $m < \log \mu_T^{-1}(m)+\pi$ for $m = \mu_T(t) > \pi$, F. G. Avkhadiev and K.-J. Wirths [3] deduced a sharp explicit form of the above theorem.

Our main goal in the present talk is to extend the Teichmüller theorem to higher dimensions (a main problem here is that there is no analogue of the Uniformization Theorem in $\mathbb{R}^n$, $n \geq 3$). In addition, we apply this result to studying the boundary correspondence problems. We emphasize that our approach may allow us to weaken regularity or quasiconformality assumptions of the mappings. Such applications to mappings of finite directional dilatations will be also presented.

The talk is based on [4] and on our forthcoming paper.

References

[1] O. Teichmüller, *Untersuchungen über konforme und quasikonforme Abbildung.* Deutsche Math. **3** (1938), 621–678.

[2] D. A. Herron, X. Y. Liu and D. Minda, *Ring domains with separating circles or separating annuli.* J. Analyse Math. **53** (1989), 233–252.

[3] F. G. Avkhadiev and K.-J. Wirths, *Schwarz-Pick type inequalities.* Frontiers in Mathematics. Birkhäuser Verlag, Basel, 2009.

[4] A. Golberg, T. Sugawa and M. Vuorinen, *Teichmüller's theorem in higher dimensions and its applications.* Comput. Methods Funct. Theory 20 (2020), no. 3-4, 539–558.

• Hadamard product and volume integral means over spherical shell

Boban Karapetrovic

AFFILIATION / INSTITUTION: University of Belgrade, Faculty of Mathematics

EMAIL: bkarapetrovic@matf.bg.ac.rs

ABSTRACT: We investigate integral means over spherical shell of holomorphic functions in the unit ball with respect to the weighted volume measures and their relation with the weighted Hadamard product. The main result has many consequences which improve some well-known estimates related to the Hadamard product in Hardy and weighted Bergman spaces.

References

[1] B. Karapetrović, J. Mashreghi, *Hadamard convolution and area integral means in Bergman spaces.* Results Math. **75** (2020).

[2] B. Karapetrović, J. Mashreghi, *Hadamard products in weighted Bergman spaces.* J. Math. Anal. Appl. **494** (2021).

[3] B. Karapetrović, *Volume integral means over spherical shell.* Canad. Math. Bull. 1-18 (2021).

- **Approximation by modified Taylor polynomials**

Javad Mashreghi

Université Laval, Quebéc, Canada

javad.mashreghi@mat.ulaval.ca

It is known that the sequence of Taylor polynomials may diverge in the local Dirichlet spaces. However, the sequence of Fejer means is a good remedy and it converges to the initial function in the norm. Another possibility is to modify the last term of Taylor polynomials and create a convergent sequence. We study this phenomenon as an orthogonal projection to the subspace of polynomials of degree at most n.

- **The characterization of (asymmetric) dual truncated Toeplitz operators**

by **Marek Ptak** (University of Agriculture in Krakow, Poland)

Dual truncated Toeplitz operators are the restrictions of multiplications operators on L^2 on the unit circle to the orthogonal complement $K_\theta^\perp$ of model spaces K_θ for the given inner function θ. The necessary and sufficient conditions for any operator on $K_\theta^\perp$ to be a dual truncated Toeplitz is given. The asymmetric case i.e. operators between different spaces $K_\theta^\perp$ and $K_\alpha^\perp$ is also considered.

Joint work with C. Câmara, K. Kliś–Garlicka, B. Łanucha.

• Square Roots of Some Classical Operators

William T. Ross

Department of Mathematics and Computer Science / University of Richmond

wross@richmond.edu

In this joint work with Javad Mashreghi and Marek Ptak, I explore the square roots of some classical operators including the Volterra, Cesàro, Hilbert matrix, the square of the shift, certain Toeplitz operators, and compressed shifts.

• ON THE SQUEEZING FUNCTION FOR FINITELY CONNECTED PLANAR DOMAINS

Oliver Roth and Pavel Gumenyuk

Department of Mathematics, University of Würzburg, Emil Fischer Straße 40, 97074, Würzburg, Germany.

EMAIL: roth@mathematik.uni-wuerzburg.de

ABSTRACT: In a recent paper, Ng, Tang and Tsai (Math. Ann. 2020) have found an explicit formula for the squeezing function of an annulus via the Loewner differential equation. Their result has led them to conjecture a corresponding formula for planar domains of any finite connectivity stating that the extremum in the squeezing function problem is achieved

for a suitably chosen conformal mapping onto a circularly slit disk. In this paper we disprove this conjecture. We also give a conceptually simple potential– theoretic proof of the explicit formula for the squeezing function of an annulus which has the added advantage of identifying all extremal functions.

- **Clark measures for rational inner functions**

Alan Sola and Kelly Bickel & Joseph Cima

Stockholm University

ola@math.su.se

ABSTRACT

I will discuss structural properties of Clark measures associated with two-variable rational inner functions on the bidisk and their associated Clark embedding. In a restricted case, precise descriptions of these measures and operators are obtained, leading to a characterization of when the associated embeddings are isometries.

- **On the Hilbert L-matrix**

František Štampach

Faculty of Nuclear Sciences and Physical Engineering / CTU in Prague:

EMAIL: stampfra@fjfi.cvut.cz

In recent paper [1], the authors studied the norm of the so-called Hilbert L-matrix

$$L_\nu = \begin{pmatrix} \frac{1}{\nu} & \frac{1}{1+\nu} & \frac{1}{2+\nu} & \frac{1}{3+\nu} & \cdots \\ \frac{1}{1+\nu} & \frac{1}{1+\nu} & \frac{1}{2+\nu} & \frac{1}{3+\nu} & \cdots \\ \frac{1}{2+\nu} & \frac{1}{2+\nu} & \frac{1}{2+\nu} & \frac{1}{3+\nu} & \cdots \\ \frac{1}{3+\nu} & \frac{1}{3+\nu} & \frac{1}{3+\nu} & \frac{1}{3+\nu} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

regarded as an operator on $\ell^2(\mathbb{N}_0)$, for $\nu > 0$, and showed that $\|L_\nu\| = 4$, if $\nu > 1/2$ and $\|L_\nu\| > 4$, if $\nu < 1/4$. In this talk, we provide a detailed spectral analysis of the operator L_ν for $\nu \in \mathbb{R} \setminus (-\mathbb{N}_0)$. As an application, we determine the quantities

$$\inf\{\nu > 0 \mid \|L_\nu\| = 4\}$$

and

$$\|L_\nu\|, \quad \text{for } 0 < \nu < 1/2,$$

answering open problems from [1].

References

[1] L. Bouthat, J. Mashreghi, *The norm of an infinite L-matrix.* Operators and Matrices, **15** (2021), 47–58.

• Spectra of Weighted Composition Operators with Quadratic Compositional Symbol

Derek Thompson and Scott Kaschner, Jessica Doctor, Alex McFarland, Timothy Hodges

Taylor University

theycallmedt@gmail.com

In a 2014 paper by Cowen et al., spectra of weighted composition operators $T_\psi C_\varphi$ on the Hardy space H^2 are determined when φ converges uniformly under iteration on the entire open unit disk to its Denjoy-Wolff point. In a 2012 paper by Bourdon, spectral results on $T_\psi C_\varphi$ are given when φ is "essentially linear fractional." By combining these results, we find the spectra of $T_\psi C_\varphi$ when $\psi \in H^\infty$ and φ is a quadratic self-map of the disk.

References

[1] Cowen, Carl C and Ko, Eungil and Thompson, Derek and Tian, Feng, *Spectra of some weighted composition operators on H^2.* Acta Sci. Math (Szeged)

[2] Bourdon, Paul, *Spectra of some composition operators and associated weighted composition operators.* Journal of Operator Theory

• Discrete analytic Schur functions

Dan Volok

Kansas State University

danvolok@math.ksu.edu

We introduce the Schur class of functions, discrete analytic on the integer lattice in the complex plane. As a special case, we derive the explicit form of discrete analytic Blaschke factors and solve the related basic interpolation problem. This is joint work with Daniel Alpay.

References

[1] D. Alpay and D. Volok, *Discrete analytic Schur functions*. arXiv:2106.04000 [math.CV]

4.4 Special Session 4 - Infinite Dimensional Analysis and Stochastic Processes

• Supermixing and hypermixing operators

Mohammad Ansari

AFFILIATION / INSTITUTION: Azad University of Gachsaran

EMAIL: ansari.moh@gmail.com

ABSTRACT As stronger versions of the mixing property, we present the newly introduced notions of *supermixing* and *hypermixing* for continuous linear operators on topological vector spaces [1]. Let X be a topological vector space and $\mathcal{L}(X)$ be the space comprising of all continuous linear operators on X. An operator $T \in \mathcal{L}(X)$ is said to be *mixing* if, for each pair of nonempty open subsets U, V of X, there exists some $N \geq 0$ such that $T^n(U) \cap V \neq \emptyset$ for all integers $n \geq N$.

An operator $T \in \mathcal{L}(X)$ is called *supermixing* if, for each nonempty open set $U \subseteq X$,

$$\overline{\bigcup_{i=0}^{\infty} \bigcap_{n=i}^{\infty} T^n(U)} = X.$$

We say that T is *hypermixing* if, for every nonempty open subset U of X, we have

$$X \backslash \{0\} \subseteq \bigcup_{i=0}^{\infty} \bigcap_{n=i}^{\infty} T^n(U).$$

We give the hypermixing and supermixing criteria and, as an application, we fully characterize the hypermixing and supermixing weighted backward shifts on Banach spaces c_0 and ℓ^p $(1 \leq p < \infty)$.

References

[1] M. Ansari, *Supermixing and hypermixing operators.* Journal of Mathematical Analysis and applications, **498** (1), (2021).

• Cantor dynamics, C*-algebras, and K-theory

Sergey Bezuglyi

University of Iowa

sergii-bezuglyi@uiowa.edu

ABSTRACT The talk is focused on the interplay between the properties of homeomorphisms of a Cantor set and the corresponding C*-algebras and K-groups. Two principal cases, minimal and non-minimal homeomorphisms, are discussed. New results are proved in a joint paper with Zhuang Niu and Wei Sun [1] where homeomorphisms of a Cantor set with k ($k < +\infty$) minimal invariant closed (but not open) subsets are studied. Crossed product C*-algebras associated to these Cantor systems and their certain orbit-cut sub-C*-algebras are considered. In the case that $k \geq 2$, the crossed product C*-algebra has stable rank 2 and real rank zero. The image of the index map is connected to certain directed graphs arising from the Bratteli-Vershik-Kakutani models of the Cantor systems. Using this, it is shown that the image of the index map actually has rank $k-1$, and it must consist of elements vanishing under all traces (infinitesimal elements).

References

[1] Sergey Bezuglyi, Zhuang Niu, and Wei Sun. *C*-algebras of a Cantor system with finitely many minimal subsets: structures, K-theories, and the index map.* Ergodic Theory Dynam. Systems, 41 (2021), no. 5, 1296–1341.

• Operators Whose Free Distributions are Dictated by the Semicircular Law

ILWOO CHO

The semicircular law is the noncommutative counterpart of the classical Gaussian distribution (or, the normal distribution) in commutative function theory. So, it plays a key role not only in noncommutative analysis including free probability, operator algebra and operator theory, but also in related applied areas including quantum statistical physics. We starts from a question: are there any free random variables, which are not self-adjoint (and hence, not semicircular), whose distributions are followed by the semicircular law in a certain manner? In this talk, we provide a positive answer with construction and abstract-ization of such operators. We, in particular, consider certain operators induced by mutually orthogonal $|\mathbb{Z}|$-many projections and an action of the infinite abelian cyclic group $\mathbb{Z}$. We showed that such operators are in general not self-adjoint, and hence, they do not induce corresponding semicircular elements. However, their joint free distributions are followed by the semicircular law in a certain sense. It shows that there are suitably many operators, which are not semicircular, but whose (joint-)free distributions (with their adjoints) are completely characterized, or dictated by the semicircular law.

• Parseval frames generated by row co-isometries

Dorin Dutkay

University of Central Florida

EMAIL:ddutkay@gmail.com

ABSTRACT

We show how Parseval frames can be generated by iterations of row co-isometries in Hilbert spaces. In the case when one starts with Cuntz isometries, one obtains orthonormal bases. Examples include Fourier series on fractals, Walsh bases, and combitations of the two. Connections are made with random walks.

• Isometric embeddings of p-Wasserstein spaces, $p \geq 1$

György Pál Gehér, Tamás Titkos and Dániel Virosztek

University of Reading, UK

gehergyuri@gmail.com

Given a complete and separable metric space X, one defines its Wasserstein space as the collection of sufficiently concentrated Borel probability measures endowed with a metric which is calculated by means of optimal transport plans. This notion has strong connections to many flourishing areas in pure and applied mathematics, moreover, the p-Wasserstein space itself is an interesting object, being a measure theoretic analogue of L^p spaces.

Motivated by Kloeckner's result on the isometry group of the quadratic ($p = 2$) Wasserstein space $\mathcal{W}_2(\mathbb{R}^n)$ ([3]), we describe the isometry group $\mathrm{Isom}(\mathcal{W}_p(E))$ for all parameters $1 \leq p < \infty$ and for all separable real Hilbert spaces E. In the case of the real line and $p > 1, p \neq 2$, we also obtain the isometric embedding semigroup $\mathrm{IsEmb}(\mathcal{W}_p(\mathrm{R}))$.

The talk is based on the papers [1, 2].

References

[1] Gy. P. Gehér, T. Titkos, D. Virosztek, *Isometric study of Wasserstein spaces – the real line*, Trans. Amer. Math. Soc., **373** (2020), 5855–5883.

[2] Gy. P. Gehér, T. Titkos, D. Virosztek, *The isometry group of Wasserstein spaces: the Hilbertian case*, arXiv:2102.02037.

[3] B. Kloeckner, *A geometric study of Wasserstein spaces: Euclidean spaces*, Annali della Scuola Normale Superiore di Pisa - Classe di Scienze **9** (2010), 297–323.

• Euclidean Green's function for relativistic N particle system

Shaikh Gohin Samad and Wayne Polyzou

AFFILIATION / INSTITUTION:University of Iowa

EMAIL:shaikhgohin-samad@uiowa.edu

ABSTRACT: In this talk a Euclidean formulation of relativistic quantum mechanics for systems of a finite number of degrees of freedom will be discussed. Relativistic treatments of quantum theory are needed to study hadronic systems at sub-atomic distance scales. Special relativity and quantum mechanics are most naturally combined using relativistic quantum field theory. Quantum field theory is ill defined, but can be used when perturbation theory is justified. Unfortunately the interactions involving quarks are too strong for a perturbative treatment. While direct interaction approaches to relativistic quantum mechanics have proved to be useful, they have two disadvantages. One is that cluster properties are difficult to realize for systems of more than two particles. The second is that the relation to quantum field theories is indirect. Alternative formulations of relativistic quantum mechanics are possible, but it is difficult to formulate theories with all of desired properties. Euclidean formulations of relativistic quantum mechanics motivated by the Euclidean axioms of quantum field theory(Konrad Osterwalder and Robert Schrader 1974) provide an alternative representation that does not have these difficulties. More surprising, the theory can be formulated entirely in the Euclidean representation without the need for analytic continuation. In this talk Kernels for systems of N free particles of any spin are discussed.Reflection positivity are established for desired kernals. Explicit formulas for generators of the Poincaré group for any spin are constructed and shown to be self-adjoint on the Euclidean representation of the Hilbert space. The structure of correlations that preserve both the Euclidean covariance and reflection positivity are discussed.

This talk is partially based on a joint work with Wayne Polyzou.

References

[1] Gohin Shaikh Samad,Wayne N. Polyzou, *Euclidean formulation of relativistic quantum mechanics of N particles.* Physical Review C

• Duality for Representation Frames

Deguang Han

There is an abstract version of the Gabor systems duality principle for group representations, and it is known that this duality principle has some connections with the classification problem for free-group von Neumann algebras. In this talk I will revisit this general duality principle, and discuss some recent both trivial and nontrivial observations that lead to a generalization of the Wexler-Raz biorthogonality and the Fundamental Identity for Gabor representations to general group representations. Additionally, I will also discuss a duality principle connecting the "super-frames" and "muti-frames" through a commutant dual pair of group representations .

• The orbit-closed C-numerical range and majorization

Jireh Loreaux and Sasmita Patnaik

AFFILIATION / INSTITUTION: Southern Illinois University Edwardsville

EMAIL: jloreau@siue.edu

This talk will serve as an introduction to generalized numerical ranges, but will emphasize recent work on a particular generalization known as the *orbit-closed C-numerical range*, where C is a fixed trace-class operator. We establish that, when C is selfadjoint, there is a characterization of the orbit-closed C-numerical range and majorization, which leads naturally to a convexity result, generalizing the Toeplitz–Hausdorff theorem. We give a variety of other properties of the orbit-closed C-numerical range, including a description of its closure in terms of the essential numerical range, generalizing a result of Lancaster.

References

[1] Jireh Loreaux and Sasmita Patnaik, *Convexity of the orbit-closed C-numerical range and majorization.* Linear and Multilinear Algebra

[2] Jireh Loreaux and Sasmita Patnaik, *Closedness of the orbit-closed C-numerical range and submajorization.* Preprint

• Random sampling and reconstruction of concentrated signals in a reproducing kernel space

Qiyu Sun

Department of Mathematics, University of Central Florida, Orlando, FL 32816, USA. Email: qiyu.sun@ucf.edu

In this talk, I will discuss random sampling of signals concentrated on a bounded Corkscrew domain of a metric measure space, and reconstructing concentrated signals approximately from their (un)corrupted sampling data taken on a sampling set contained in the domain Ω. This talk is based on a joint paper with Yaxu Li and Jun Xian.

• Commutants of the infinite Hilbert operators

Hadi Roopaei

University of Alberta

h.roopaei@gmail.com

ABSTRACT

In this research, we introduce four classes of operators which commute with the infinite Hilbert operators and as an application we find the bounds of these operators on some sequence spaces. Moreover we obtain the ℓ_p-norm of two of these operators.

References [1] G. Bennett, Factorizing the classical inequalities, Mem. Amer. Math. Soc. 576 (1996). [2] G. Bennett, Lower bounds for matrices II, Canad. J. Math. 44 (1992), 54-74. [3] D. Foroutannia, H. Roopaei, Bounds for the norm of lower triangular matrices on the Cesàro weighted sequence space, J. Inequal. Appl., 2017(67)(2017), 1-11. [4] G. H. Hardy, An inequality for Hausdorff means, J. London Math. Soc., 18 (1943), 46-50. [5] G. H. Hardy, Divergent series, Oxford University Press, 1973. [6] G. H. Hardy, J. E. Littlewood, G. Polya, Inequalities, 2nd edition, Cambridge University Press, Cambridge, 2001. [7] H. Kizmaz, On certain sequence spaces I, Canad. Math. Bull., 25(2)(1981), 169-176. [8] L. Bouthat, J. Mashreghi, The norm of an infinite L-matrix, Oper. Matrices, 2020, accepted [9] H. Roopaei, D. Foroutannia, The norm of matrix operators on Cesàro weighted sequence space, Linear Multilinear Algebra, 67(1) (2019), 175-185. [10] H. Roopaei, Norm of Hilbert operator on sequence spaces, J. Inequal. Appl., 117: 2020, (2020). [11] H.

Roopaei, Factorization of Cesàro and Hilbert matrices based on generalized Cesàro matrix, Linear Multilinear Algebra, 68 (1) (2020), 193-204. [12] H. Roopaei, Bounds of operators on the Hilbert sequence space, Concr. Oper. (7) (2020), 155–165. [13] H. Roopaei, D. Foroutannia, M. İlkhan, E. E. Kara, Cesàro spaces and norm of operators on these matrix domains, Mediterr. J. Math., 17, 121 (2020) published online. [14] H. Roopaei, F. Başar, On the spaces of Cesàro absolutely p-summable, null and convergent sequences, Math. Methods Appl. Sci., (2020), accepted. [15] H. Roopaei, A study on Copson operator and its associated sequence space, J. Inequal. Appl., (2020), 2020:120 published online. [16] H. Roopaei, Factorization of the Hilbert matrix based on Cesàro and Gamma matrices, Results Math.,75 (1) 3, 2020.

• Bratteli-Vershik models for substitutions on a infinite alphabet.

Shrey Sanadhya

University of Iowa

We consider substitutions on countably infinite alphabet as Borel dynamical systems and build their Bratteli-Vershik models. We prove two versions of Rokhlin's lemma for such substitution dynamical systems. Using the Bratteli-Vershik model we give an explicit formula for a shift-invariant measure (finite and infinite) and provide a criterion for this measure to be ergodic (or uniquely ergodic). This is joint work with Sergii Bezuglyi and Palle Jorgensen.

• Orthogonality and Gateaux derivative of C^* norm

Sushil Singla

AFFILIATION / INSTITUTION: Shiv Nadar University, Tehsil Dadri, Greater Noida, Uttar Pradesh, India.

EMAIL: ss774@snu.edu.in

ABSTRACT

Birkhoff-James orthogonality is a generalization of Hilbert space orthogonality to normed spaces. In a given normed space V, an element v is said to be Birkhoff-James orthogonal to a subspace W if $\|v\| \leq \|v-w\|$ for all $w \in$

W. Consider the function $g(\lambda) = \|v + \lambda w\|$, mapping $\mathbb{F}$ into $\mathbb{R}_+$. Since $\|\cdot\|$ is a convex function, $\lim_{t\to 0^+} \dfrac{g(t)-g(0)}{t}$ always exists, known as Gateaux derivative of $\|\cdot\|$ at v. For a C^*-algebra $\mathcal{A}$, we shall give an expression for the Gateaux derivative of the C^* norm in terms of states on $\mathcal{A}$. As a consequence, we will obtain a characterization of orthogonality of an element of an ideal of $\mathcal{A}$ to a subspace of $\mathcal{A}$. We shall also give a few others applications including a characterization of smooth points of the space of bounded operators on a Hilbert space.

References

[1] P. Grover, S. Singla, *Birkhoff-James orthogonality and applications : A survey.* Operator Theory, Functional Analysis and Applications, Birkhäuser Basel, vol. 282, accepted.

[2] P. Grover, S. Singla, *Best approximations, distance formulas and orthogonality in C^*-algebras.* J. Ramanujan Math. Soc., 36, No. 1 (2021), 85–91.

[3] S. Singla, *Gateaux derivative of the C^* norm.* communicated.

• Data Dimension Reduction Using Principal Component Analysis

Myung-Sin Song and Palle Jorgensen, Sooran Kang, James Tian

AFFILIATION / INSTITUTION: Southern Illinois University Edwardsville

EMAIL: msong@siue.edu

ABSTRACT

In linear data case, Principal Component Analysis is used for data dimension reduction. In nonlinear data dimension reduction , kernel-Principal Component Analysis is used instead with manifold and feature space transforms. The results extend earlier work for probabilistic Karhunen-Loève transforms on compression of wavelet images which were algorithms for optimization, selection of efficient bases, or components, which serve to minimize entropy and error; and hence to improve digital representation of images, and hence of optimal storage, and transmission. Several new theorems for data-dimension reduction will be presented, and with the use of frames in Hilbert space, and a new Hilbert-Schmidt analysis, we identify when a choice of Gaussian kernel is optimal.

- **A Kaczmarz algorithm for sequences of projections**

James Tian

Mathematical Reviews

jft@ams.org

We discuss connections between the classical Kaczmarz algorithm, construction of frames, and convergence of infinite product of projections in Hilbert spaces.

4.5 Special Session 5 - Laplacian Eigenfunctions – Counts, Morphologies and Statistics

• Universality of nodal count distribution in large metric graphs

Lior Alon , **Ram Band and Gregory Berkolaiko**

AFFILIATION / INSTITUTION: Institute for Advanced Study, Princeton

EMAIL: lalon@ias.edu

An eigenfunction of the Laplacian on a metric (quantum) graph has an excess number of zeros due to the graph's non-trivial topology. This number, called the nodal surplus, is an integer between 0 and the rank β of the graph's fundamental group. We study the distribution of the nodal surplus values in the countably infinite set of the graph's eigenfunctions. We conjecture that this distribution converges to Gaussian for any sequence of graphs of growing β. We prove this conjecture for several special graph sequences and test it numerically for a variety of well-known graph families. Accurate computation of the distribution is made possible by a formula expressing the nodal surplus distribution as an integral over a high-dimensional torus.

References

[1] Lior Alon , Ram Band and Gregory Berkolaiko, *Universality of nodal count distribution in large metric graphs.* Preliminary report.

• Neumann domains

Ram Band

AFFILIATION / INSTITUTION: Technion

EMAIL: ramband@technion.ac.il ABSTRACT

The nodal set of a Laplacian eigenfunction forms a partition of the underlying manifold or graph. Another natural partition is based on the gradient vector field of the eigenfunction (on a manifold) or on the extremal points of the eigenfunction (on a graph). The submanifolds (or subgraphs) of this partition are called Neumann domains (you may guess

the reason for this name, and it would also be mentioned in the talk ;) We present results concerning Neumann domains on manifolds and on graphs.

The talk is based on joint works with Lior Alon, Graham Cox, Sebastian Egger, David Fajman and Alexander Taylor.

References

[1] AUTHORS, *TITLE*. JOURNAL

• A Friedland-Hayman inequality for convex subsets of the sphere

Thomas Beck and David Jerison, Sarah Raynor

Fordham University

tbeck7@fordham.edu

The Friedland-Hayman inequality provides a lower bound on the first Dirichlet eigenvalues of complementary subsets of the sphere. In this talk, we will describe a version of this inequality for geodesically convex subsets of the sphere with mixed Dirichlet-Neumann boundary conditions. The proof, which uses a variant of Caffarelli's contraction theorem for the Brenier optimal transport mapping, also allows us to characterize the case of equality. In particular, we will show that equality is attained precisely when the corresponding eigenfunctions are the restrictions to the sphere of linear functions, vanishing on a half-space.

References

[1] T. Beck, D. Jerison, S. Raynor, *Two-phase free boundary problems in convex domains*. The Journal of Geometric Analysis

[2] T. Beck, D. Jerison, *The Friedland-Hayman inequality and Caffarelli's contraction theorem*. arXiv preprint: arXiv:2102.00571

• Level sets and critical points of eigenfunctions

Philippe Charron

AFFILIATION / INSTITUTION: Technion, Haifa, Israel

EMAIL: philippe.ch@campus.technion.ac.il

ABSTRACT

In this talk I will discuss recent results regarding the geometrical and topological properties of level sets of eigenfunctions of the Laplacian on manifolds, as well as the distribution of their critical points. I will also outline a few constructions of pathological metrics on surfaces as well as instability results that will highlight some of the difficulties of trying to give bounds in the general case.

This will include recent works in collaboration with Pierre Bérard and Bernard Helffer.

• Nodal deficiency via equipartitions and Dirichlet-to-Neumann maps

Graham Cox

Memorial University

gcox@mun.ca

Courant's nodal domain theorem, which says the nth Laplacian eigenfunction has at most n nodal domains, is almost always a strict inequality. The extent to which it fails to be sharp is measured by the nodal deficiency. Despite much study, this quantity is still not very well understood except in highly symmetric cases. However, in the last decade two general formulas for the nodal deficiency were established.

The first was given in 2012 by Berkolaiko, Kuchment and Smilansky, using an energy functional defined on the space of equipartitions of the domain. More recently, with Jones and Marzuola, I obtained a formula for the nodal deficiency in terms of a two-sided Dirichlet-to-Neumann map defined on the nodal set.

After reviewing both of these results, I will describe new work (with Gregory Berkolaiko, Yaiza Canzani and Jeremy Marzuola) that demonstrates a direct connection between these seemingly different approaches to nodal deficiency. Among other things, it gives a method for using the Dirichlet-to-Neumann map to calculate eigenfunctions for the Hessian of the equipartition energy, and provides insight into the theory of spectral minimal partitions.

• Spectral flow for pair compatible equipartitions

Bernard Helffer

We show that a recent spectral flow approach proposed by Berkolaiko–Cox–Marzuola for analyzing the nodal deficiency of the nodal partition associated to an eigenfunction can be extended to more general not necessarily bipartite partitions. To be more precise, we work with spectral equipartitions that satisfy a pair compatible condition. Nodal partitions and spectral minimal partitions are examples of such partitions.

Along the way, we discuss, using former collaborations with M. and T. Hoffmann-Ostenhof, M. Owen, V. Bonnaillie, S. Terracini, different approaches to the Dirichlet-to-Neumann operators but will mainly discuss an approach based on the construction of an Aharonov-Bohm operator.

This work is in collaboration with M. Persson Sundqvist (University of Lund).

• Boundedness of the number of nodal domains of equivariant eigenfunctions

Junehyuk Jung and **Steve Zelditch**

Brown University

EMAIL:junehyuk_jung@brown.edu

In this talk, I'm going to explain my work with Steve Zelditch, where we prove that, when M is a principle S^1-bundle equipped with a generic Kaluza-Klein metric, the nodal counting of eigenfunctions is typically 2, independent of the eigenvalues. Note that principle S^1-bundle equipped with a Kaluza-Klein metric never admits ergodic geodesic flow. This, for instance, contrasts the case when (M, g) is a surface with non-empty boundary with ergodic geodesic flow (billiard flow), in which case the number of nodal domains of typical eigenfunctions tends to $+\infty$. I will also present an orthonormal eigenbasis of Laplacian on a flat 3-torus, where every non-constant eigenfunction has exactly two nodal domains. In particular, this tells us that the number of nodal domains could be uniformly bounded independent of the eigenvalue.

References

[1] Junehyuk Jung and Steve Zelditch, *Boundedness of the number of nodal domains for eigenfunctions of generic Kaluza-Klein 3-folds.* Ann. Inst. Fourier. (2020)

• SPECTRAL MINIMAL PARTITIONS OF METRIC GRAPHS

MATTHIAS HOFMANN, JAMES KENNEDY, DELIO MUGNOLO, and MARVIN PLÜMER

AFFILIATION / INSTITUTION: Group of Mathematical Physics, University of Lisbon

EMAIL: jbkennedy@fc.ul.pt

We give a brief overview of the theory of spectral minimal partitions of metric graphs developed recently in [2,3,4]. Such partition problems, which are closely related to the surgical operation of *cutting* a graph, allow far more freedom than their domain counterparts, as one can choose both where cuts are permitted and what types of vertex conditions (e.g. Dirichlet or standard) one imposes at the cuts.

We will illustrate how the spectral minimal partitions built on the eigenvalues of the Laplacian with Dirichlet or standard conditions are good proxies for the eigenvalues of the standard Laplacian on the whole graph in a number of ways. This involves not just bounds and Weyl-type asymptotics for the spectral minimal energies themselves, but also interlacing inequalities strongly reminiscent of bounds on the difference between the number of nodal and Neumann domains of the whole graph eigenfunctions [1, Proposition 11.2].

References

[1] L. Alon, R. Band, M. Bersudsky and S. Egger, *Neumann domains on graphs and manifolds*. Chapter 10 in M. Keller, D. Lenz and R. K. Wojciechowski (eds.), Analysis and Geometry on Graphs and Manifolds. London Mathematical Society Lecture Note Series, vol. 461, 2020.

[2] M. Hofmann and J. B. Kennedy, *Interlacing and Friedlander-type inequalities for spectral minimal partitions of metric graphs*. Preprint (2021), arXiv:2102.07585.

[3] M. Hofmann, J. B. Kennedy, D. Mugnolo and M. Plümer, *Asymptotics and estimates for spectral minimal partitions of metric graphs*. To appear in Integral Equ. Oper. Theory, preprint arXiv:2007.01412.

[4] J. B. Kennedy, P. Kurasov, C. Léna and D. Mugnolo, *A theory of spectral partitions of metric graphs*. To appear in Calc. Var. PDE, preprint arXiv:2005.01126.

• SPECTRAL SHIFT VIA “LATERAL” PERTURBATION

Gregory Berkolaiko and Peter Kuchment

Mathematics Department, Texas A& M University

berko@math.tamu.edu, kuchment@math.tamu.edu

ABSTRACT We consider a compact perturbation $H_0 = S + K_0^* K_0$ of a self-adjoint operator S with an eigenvalue λ° below its essential spectrum and the corresponding eigenfunction f. The perturbation is assumed to be “along” the eigenfunction f, namely $K_0 f = 0$. The eigenvalue λ° belongs to the spectra of both H_0 and S. Let S have σ more eigenvalues below λ° than H_0; σ is known as the spectral shift at λ°.

We now allow the perturbation to vary in a suitable operator space and study the continuation of the eigenvalue λ° in the spectrum of $H(K) = S + K^* K$. We show that the eigenvalue as a function of K has a critical point at $K = K_0$ and the Morse index of this critical point is the spectral shift σ. A version of this theorem also holds for some non-positive perturbations.

References

[1] G. Berkolaiko and P. Kuchment *Spectral shift via "lateral" perturbation*. J. Spectral Theory, to appear

- **Pleijel-type upper bound for the nodal count**

Corentin Léna and Philippe Charron

AFFILIATION / INSTITUTION: University of Neuchâtel

EMAIL: corentin.lena@unine.ch

For many self-adjoint differential operators, Courant's nodal theorem tells us that an eigenfunction associated with eigenvalue number k has at most k nodal domains. Å. Pleijel showed in 1956 that for the Dirichlet-Laplacian on a given planar domain, equality can be reached only for a finite number of eigenvalues.

Pleijel's proof actually gives an asymptotic upper bound of the number of nodal domains. It has been extended afterwards to other geometric settings, boundary conditions and operators. In recent years several generalizations and refined versions have been obtained, and a large number of special cases analyzed. In the continuity of these results, I will show that the upper bound holds for a large class of Schrödinger operators. This is joint work with Philippe Charron.

- **Sign of Laplace eigenfunctions and quasi-symmetry conjecture.**

A.Logunov

Consider a sequence of real Laplace eigenfunctions on a closed Riemannian manifold. The first eigenfunction is constant, which we will not be considered further, and all the rest change sign.

We will discuss a recent result on the distribution of their sign for two-dimensional manifolds. The total area, where the eigenfunction is positive, is comparable to the area, where the eigenfunction is negative.

Based on a joint work in progress with F.Nazarov.

• On Dirichlet Laplace eigenfunctions in Lipschitz domains

Eugenia Malinnikova and Alexander Logunov, Nikolai Nadirashvili, Fedor Nazarov

Stanford University

EMAIL: eugeniam@stanford.edu

We consider bounded domains in the Euclidean space with Lipschitz boundary and locally small Lipschitz constant. We proof the sharp upper bound for the area of the nodal sets of Dirichlet Laplace eigenfunctions in such domains. One of our tools is the analysis of the frequency function of a harmonic function vanishing on a part of the boundary.

References

[1] A. Logunov, E. Malinnikova, N. Nadirashvili, F. Nazarov *The sharp upper bound for the area of the nodal sets of Dirichlet Laplace eigenfunctions.* arXiv:2104.09012

• A bound for the eigenvalue counting function for Krein—von Neumann and Friedrichs extensions of elliptic operators

Selim Sukhtaiev

Auburn University

EMAIL: szs0266@auburn.edu

In this talk, I will discuss a bound for the eigenvalue counting function (for strictly positive eigenvalues) for Krein—von Neumann and Friedrichs extensions of higher-order elliptic operators. The latter are particular self-adjoint extensions of minimally defined, positive integer powers of elliptic operators on arbitrary open, bounded sets. The bound shows the correct high-energy power law behavior familiar from Weyl asymptotics. This talk is based on joint work with M. Ashbaugh, F. Gesztesy, A. Laptev, and M. Mitrea.

- **A new approach to the hot spots conjecture**

Jonathan Rohleder

Stockholm University

jonathan.rohleder@math.su.se

It is a conjecture going back to J. Rauch (1974) that the hottest and coldest spots in an insulated homogeneous medium such as an insulated plate of metal should converge to the boundary, for "most" initial heat distributions, as time tends to infinity. This so-called hot spots conjecture can be phrased alternatively as follows: the eigenfunction(s) corresponding to the first non-zero eigenvalue of the Neumann Laplacian on a Euclidean domain should take its maximum and minimum on the boundary only. This has been proven to be false for certain domains with holes, but it is by now known to hold for certain classes of simply connected or convex planar domains. In this talk we provide an entirely new approach to the conjecture, which proves it for a large class of simply connected planar domains.

- **On the defect ("signed area") of toral Laplace eigenfunctions and exponential sums.**

Pär Kurlberg, Igor Wigman and Nadav Yesha

AFFILIATION / INSTITUTION: King's College London

EMAIL: igor.wigman@kcl.ac.uk

ABSTRACT

The defect (also known as "signed area") of a real-valued function defined on a two-dimensional domain is the difference between its positive and negative regions. We are interested in the defect of toral Laplace eigenfunctions (exponential sums) restricted to Planck-scale shrinking subdomains ("shrinking balls"). It is proved that, under a flatness assumption on the exponential sums, the defect asymptotically vanishes on the set of balls centres of almost full measure, for a generic sequence of energy levels. To establish our results we start from Bourgain's derandomization technique, and also borrow the Integral-Geometric sandwich from Nazarov-Sodin, and also invoke other techniques.

References

[1] Pär Kurlberg, Igor Wigman and Nadav Yesha *The defect of toral Laplace eigenfunctions and Arithmetic Random Waves* arXiv:2006.11644

- **Spherical harmonics on spheres S^N of dimension N with exactly 2 nodal domains**

Steve Zelditch and Junehyuk Jung

Department of mathematics / Northwestern University:

EMAIL:zelditch@math.northwestern.edu

Many years ago, H. Lewy made an ingenious construction of eigenfunctions on S^2 of arbitrarily high eigenvalue with just two nodal domains. In recent work, Junehyuk Jung and I showed that on S^3 there are many such eigenfunctions, i.e. for each degree N and $0 < |m| < N$, there is a subspace H_N^m of dimension N where the real parts of random eigenfunctions transforming by $e^{im\theta}$ under a natural circle S^1 action have a single nodal component and exactly 2 nodal domains. This is a special case of a general result on 3-manifolds which have a free S^1 action. In recent work, the results are generalized to all dimensions. In particular, the results hold for random 'equivariant spherical harmonics' on general odd $(2n+1)$-dimensional spheres. There exist S^1 actions on even dimensional spheres but they are not free and the equivariant eigenfunctions have very different nodal sets.

References

[1] J. Jung, Junehyuk and S. Zelditch, Boundedness of the number of nodal domains for eigenfunctions of generic Kaluza-Klein 3-folds. Ann. Inst. Fourier (Grenoble) 70 (2020), no. 3, 971-1027

[2] J. Jung and S. Zelditch *Topology of the nodal set of random equivariant spherical harmonics on S^3* . IMRN, to appear (arXiv:1908.00979)

[3] J. Jung and S. Zelditch, 2-nodal domain theorems for higher dimensional circle bundles (in preparation).

4.6 Special Session 6 - Passive and Dissipative Linear Systems

• THE INFINITE-DIMENSIONAL CONTINUOUS-TIME STANDARD AND STRICT BOUNDED REAL LEMMA

JOSEPH A. BALL, MIKAEL KURULA, and SANNE TER HORST

VIRGINIA TECH (BLACKSBURG), ABO AKADEMI (ABO), NORTHWEST UNIVERSITY (POTCHEFSTROOM)

joball@math.vt.edu, Mikael.Kurula@abo.fi, Sanne.TerHorst@nwu.ac.za

ABSTRACT

It is well known that a proper rational matrix function $F(\lambda)$ can be presented in realization form $F(\lambda) = F_\Sigma(\lambda) = D + C(\lambda - A)^{-1}B$ where the system j(or colligation) matrix Σ has the form $\Sigma = \left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right] : \left[\begin{smallmatrix} X \\ U \end{smallmatrix}\right] \to \left[\begin{smallmatrix} X \\ Y \end{smallmatrix}\right]$ for finite-dimensional linear spaces U (the input space), X (the state space), and Y (the output space). The classical Bounded Real Lemma characterizes in terms of the matrices A, B, C, D as to when it is the case that the associated transfer function $F_\Sigma(\lambda)$ is in the Schur-class of the right-half plane, i.e., when is it the case that $\sup_{\lambda \in \mathbb{C}_+} \|F(\lambda)\| \le 1$. The answer (now called the Kalman-Yakubovich-Popov (KYP) lemma) is: *there should exists a positive definite matrix H satisfying the KYP-inequality:* $\left[\begin{smallmatrix} HA+A^*H+C^*C & HB+C^*D \\ B^*H+D^*C & D^*D-I \end{smallmatrix}\right] \preceq 0$. The associated positive real-valued function $x \mapsto \langle Hx, x \rangle$ on the state-space gives rise to a storage function in the approach of Willems [4]. A variant is the *strict Bounded Real Lemma*, first explored by Petersen-Andeerson-Jonkheere [2] in their quest for a elementary state-space solution of the standard problem of H^∞-controlwhere one seeks a characterization as to when $sup_{\lambda \in \mathbb{C}_+} \|F(\lambda)\| < 1$.

This talk reports on on-going work of the speaker with Mikael Kurula and Sanne ter Horst on extensions of these ideas to the non-rational-function/infinite-dimensional-system context. There is still a notion of realization $F(\lambda) = F_\Sigma(\lambda) = D + C(\lambda - A)^{-1}B$ for a contractive-operator-valued holomorphic funtion on the right half-plane, but the operators A, B, C, D are unbounded in various ways (see [3]) and the positive definite operator H may be unbounded as well, requiring a more delicate formulation as to what is meant by a solution H of the KYP inequality. Nevertheless, by using these tools from infinite-dimensional operator theory and continuous-time linear systems, we afre able to adapt the recent analysis for the discrete-time-linear-system/Schur-class-over-the-unit-disk setting [1] to get reasonably complete analogous results for the continuous-time half-plane setting.

References

[1] J.A. Ball, *Standard versus strict bounded real lemma with infinite-dimensional state space II: The storage function approach*, in: The Diversity and Beauty of Applied Operator Theory, pp. 1-50, Oper. Theory Adv. Appl. **268**, Birkhäuser/Cham, 2018.

[2] I.R. Peteresen, B.D.O. Anderson, and E.A. Jonckheere, *A first principles solution to the non-singular H^∞ control problem*, Internat. J. Robust Nonlinear Control **1** (1991), 171-185.

[3] O.J. Staffans, *Well-posed Linear Systems*, Cambridge University Press, 2005.

[4] J.C. Willems, *Dissipative dynamical systems Part II: Linear systems with quadratic supply rates*, Arch. Rational Mech. Anal. **45** (1972), 352-393.

• THE SPECTRAL DECOMPOSITION OF A LINEAR RELATION

SLAVISA DJORDJEVIĆ and IVÁN MOISES ROQUE TLATELPA

AFFILIATION / INSTITUTION: BENEMERITA UNIVERSIDAD AUTONOMA DE PUEBLA

EMAIL: slavdj@fcfm.buap.mx

Let X and Y be complex Banach spaces. An arbitrary linear subspace $\mathcal{A} \subset \mathcal{X} \times \mathcal{Y}$ is called a linear relation between X and Y. If $\mathcal{A}$ is closed in $X \times Y$, then it is called a closed linear relation.

Let M be a closed subspace of X. In literature, we have different approach to define the restriction of a linear relation to a subspace M. Following Cross ([1]), for a linear relation $\mathcal{A} \subset \mathcal{X} \times \mathcal{Y}$ given by its graph

$$Gr(A) := \{(x, y) \in X \times Y \,:\, x \in Dom(A),\, y \in Ax\},$$

the restriction of $\mathcal{A}$ to a subspace M is defined by the graph

$$Gr(A_M) := \{(x, y) \in X \times Y \,:\, x \in Dom(A) \cap M,\, y \in Ax\}.$$

Another way to define the restriction of a linear relation we can find in the work of Lajnef and Mnif ([3]):

$$Gr(A_M) := Gr(A) \cap (M \times M).$$

Both of previous definitions does not give as a good tools that discuss about an invariant subspace for a linear relation. For this we need another kind of definitions of a restriction of a linear relation to a subspace M in X. In this talk we will present a definition of an invariant subspace for a linear relation given by Baskakov and Chernyshov ([2]). Using this definition, we will be able to give a decomposition of a linear relation trought a a direct sum decomposition of the space X by invariant subspaces. In this way we can decompose the spectrum of a linear relation like as the union of the spectrums of it restrictions.

References

[1] R. Cross, *Multivalued Linear Operators.* 1st ed. Dekker, 1998.

[2] A. G. Baskakov and K.I. Chernyshov, *Spectral analysis of linear relations and degenerate operator semigroups.* Sbornik Mathematics 193 (2002), 1573–1610.

[3] M. Lajnef and M. Mnif, *Isolated spectral points of a linear relation.* Monatshefte für Mathematik 191 (2020), 595-614.

• On sufficient density conditions for interpolation and sampling in a weighted Hilbert Bargmann-Fock space

Mohammed El Aïdi

Universidad Nacional de Colombia, Bogota, Colombia

melaidi@unal.edu.co

We provide geometric sufficient conditions for discrete points to be an interpolating and sampling sequence for a weighted Hilbert Bargmann-Fock space comprised by square integrable entire functions with respect to a measure given in terms of a weight which is a plurisubharmonic function on several complex variables space.

• Some algebraic aspects of quantum games

William Helton

University of California at San Diego

helton@math.ucsd.edu

ABSTRACT

The last two decades produced a substantial noncommutative (in the free algebra) real and complex algebraic geometry. The aim of the subject is to develop a systematic theory of equations and inequalities for noncommutative polynomials of operator variables. The talk will focus on a few topics which bear on quantum games.

• The Weyl matrix balls corresponding to the matricial truncated Hamburger moment problem

Kirstein

The main goal of the paper is to determine the Weyl matrix balls associated with an arbitrary matricial truncated Hamburger moment problem. For the special case of a non-degenerate matricial truncated Hamburger moment problem the corresponding Weyl matrix balls were computed by I. V. Kovalishina in the framework of V. P. Potapov's method of 'Fundamental matrix inequalities'.

• Passive Linear Time-invariant Systems Characterization through Structure

Izchak Lewkowicz

School of Electrical and Computer Engineering
Ben-Gurion University of the Negev, Beer-Sheva, Israel

izchak@bgu.ac.il

Passivity is a basic physical property. We here show that the family linear time invariant *passive* may be characterized by the structure of the whole set:

Passive linear time-invariant systems and matrix-convexity	
discrete-time	continuous-time
a maximal set closed under products of its elements	a cone closed under inversion and maximal non-singular/analytic

• Modulation spaces associated to tensor products of amalgam spaces

Stevan Pilipović S. Pilipović, Department of Mathematics and Informatics, University of Novi Sad, Trg Dositeja Obradovića 4, 21000 Novi Sad, Serbia, stevan.pilipovic@dmi.uns.ac.rs

The modulation spaces introduced by Feichtinger and later analysed by many authors are here associated to a Banach space X, laying between the spaces $\mathcal{S}$ and $\mathcal{S}'$. They consist of all tempered distributions in $\mathcal{S}'(\mathcal{R}^d)$ whose images under the short-time Fourier transform belong to X. Unlike $L^{p,q}_\eta$, the space X does not need to be solid and this generalized framework allows one to consider a wide variety of modulation spaces $\mathcal{M}[X]$. In the joint paper with H. Feichtinger and B. Prangoski, we identify the modulation spaces associated to tensor products of amalgam spaces having a large class of Banach spaces as their local component. As consequences of the main results, we describe the modulation spaces associated to tensor products of various L^p spaces.

• When Kalton and Peck met Fourier

Alberto Salguero Alarcón

Universidad de Extremadura, Badajoz, Spain

salgueroalarcon@unex.es

A twisted sum of two Banach spaces X and Y is another space Z containing Y as a subspace so that $Z/Y = X$. One of the paramount examples of twisted sums is the Kalton and Peck space Z_2, which is a non-Hilbert twisted sum of ℓ_2 with itself. This talk employs Fourier analysis, together with the fact that Z_2 inherits an ℓ_∞-module structure from ℓ_2, to obtain new twisted sums. Precisely, we construct a wide range of twisted sums of L_p-spaces which possess an L_1-module structure inherited from the L_p's. This is part of a joint work with F. Cabello Sánchez.

References

[1] F. Cabello Sánchez and A. Salguero Alarcón, *When Kalton and Peck met Fourier*; currently available on https://arxiv.org/abs/2101.11561.

[2] N. J. Kalton and N. T. Peck; *Twisted sums of sequence spaces and the three space problem*, Trans. Amer. Math. Soc, 1979 (255), pp. 1-30

• The convex invertible cone structure of positive real odd rational matrix functions

Sanne ter Horst and **Alma Naudé**

AFFILIATION / INSTITUTION: North West University

EMAIL: sanne.terhorst@nwu.ac.za

Positive real odd matrix functions, often referred to as positive real lossless matrix functions, play an important role in many applications in multi-port electrical systems. In this paper we present closer analogues to some of the known results for the scalar, one-port, case in the multi-port setting. Specifically, we determine necessary and sufficient conditions for the well studied partial fraction formula to represent functions in the class of positive real odd matrix functions, and explicit minimal state space realization formulas for the inverse (admittance) of a function in this class, which itself is also a positive real odd matrix function. Doing so, enables us to provide a partial analogue of the pole-zero interlacing behavior from the scalar case. The talk is based on [1].

References

[1] Sanne ter Horst and Alma Naudé, *The convex invertible cone structure of positive real odd rational matrix functions.* Operators and Matrices 15 (2021), 357–379

4.7 Special Session 7 - Operator Theory and Interpolation in Several Complex Variables

• Sharp width asymptotics in spaces of holomorphic functions

Oscar F. Bandtlow and Stéphanie Nivoche

Queen Mary University of London

o.bandtlow@qmul.ac.uk

Given a domain D in $\mathbb{C}^n$ and K a compact subset of D, the set $\mathcal{A}_K^D$ of all restrictions of functions holomorphic on D the modulus of which is bounded by 1 is a compact subset of the Banach space $C(K)$ of continuous functions on K. The sequence $(d_m(\mathcal{A}_K^D))_{m\in\mathbb{N}}$ of Kolmogorov m-widths of $\mathcal{A}_K^D$ provides a measure of the degree of compactness of the set $\mathcal{A}_K^D$ in $C(K)$ and the study of its asymptotics has a long history, essentially going back to Kolmogorov's work on ϵ-entropy of compact sets in the 1950s. In the 1980s Zakharyuta showed that for suitable D and K the asymptotics

$$\lim_{m\to\infty} \frac{-\log d_m(\mathcal{A}_K^D)}{m^{1/n}} = 2\pi \left(\frac{n!}{C(K,D)}\right)^{1/n}, \tag{5}$$

where $C(K,D)$ is the Bedford-Taylor relative capacity of K in D is implied by a conjecture, now known as Zakharyuta's Conjecture, concerning the approximability of the regularised relative extremal function of K and D by certain pluricomplex Green functions. Zakharyuta's Conjecture was proved by Nivoche in 2004 thus settling (5) at the same time.
In this talk I will outline a strategy for a new proof of the asymptotics (5) for D strictly hyperconvex and K non-pluripolar which does not rely on Zakharyuta's Conjecture. Instead it is possible to proceed more directly by a two-pronged approach establishing sharp upper and lower bounds for the Kolmogorov widths. The lower bounds follow from concentration results of independent interest for the eigenvalues of a certain family of Toeplitz operators, while the upper bounds follow from an application of the Bergman-Weil formula together with an exhaustion procedure by special holomorphic polyhedra.

References

[1] O.F. Bandtlow and S. Nivoche *New solution of a problem of Kolmogorov on width asymptotics in holomorphic function spaces.* To appear in *J. Eur. Math. Soc.*

• Analytic functionals for Popescu's multivariate disc algebra

Raphaël Clouâtre and Robert Martin and Edward Timko

University of Manitoba

raphael.clouatre@umanitoba.ca

Classically, a theorem of F. and M. Riesz states that a measure on the circle annihilating all polynomials must necessarily be absolutely continuous with respect to Lebesgue measure. Equivalently, this can be viewed as a description of the functionals on $C(\mathbb{T})$ that annihilate the disc algebra – the so-called *analytic* functionals.

In this talk, I will investigate analytic functionals for Popescu's multivariate version of the disc algebra. It is still true, once interpreted appropriately, that analytic functionals never have a singular part in this more general setting. Unlike the classical univariate case however, the issue of extending analytic functionals weak-$*$ continuously is more delicate, and is intertwined with the so-called universal structure projection.

• Noncommutative Hermite interpolation

Jurij Volčič, Igor Klep and Victor Vinnikov

Texas A&M University

volcic@tamu.edu

ABSTRACT

Free analysis is a quantization of the usual function theory much like operator space theory is a quantization of classical functional analysis. Basic objects of free analysis are noncommutative functions, which are maps on tuples of matrices of all sizes that preserve direct sums and similarities. This talk addresses the following interpolation question: given an analytic noncommutative function f, a number L and a finite set of matrix points S, does there exist a noncommutative polynomial p such that f and p agree at S up to noncommutative derivatives of order L? A positive answer is given in case S consists of semisimple matrix points. The scope of this result is illustrated with several examples, and its consequences for the structure of analytic noncommutative germs are described.

References

[1] I. Klep, V. Vinnikov, J. Volčič, *Local theory of free noncommutative functions: germs, meromorphic functions and Hermite interpolation.* Trans. Amer. Math. Soc. 373 (2020) 5587–5625.

• Positivity of noncommutative polynomials with traces

Igor Klep

University of Ljubljana, Faculty of Mathematics and Physics, Department of Mathematics

igor.klep@fmf.uni-lj.si

Trace polynomials are polynomials in noncommuting variables and traces of their products. They can be naturally evaluated in matrix tuples or in finite von Neumann algebras. While originating in invariant theory as equivariant maps between tuples of matrices, trace polynomials more recently received attention in operator algebra, free probability and quantum information theory. This talk addresses positivity of their evaluations and presents novel Positivstellensätze (=algebraic certificates for positivity) in terms of sums of squares and traces of sums of squares. A highlight is a Hilbert 17th-like theorem addressing positive univariate trace polynomials, where Hankel matrices and the moment problem are central.

The talk is based on joint works with Jurij Volčič, Victor Magron and James Pascoe.

• Factorization of positive Toeplitz operators on Fock space

Robert T.W. Martin

University of Manitoba

Robert.Martin@umanitoba.ca

The Hardy space, H^2, is the Hilbert space of analytic functions in the complex unit disk with square-summable Taylor series coefficients at the origin. The Hardy space embeds isometrically into L^2 of the circle by taking non-tangential boundary limits of functions in H^2 almost everywhere with respect to Lebesgue measure. A bounded operator on H^2 is called a *Toeplitz operator* if it is the compression of a multiplication operator

on L^2 of the circle to H^2. A positive Toeplitz operator, T, is said to be *factorizable* if one can find a bounded analytic function in the disk, h, so that $M_h^* M_h = T$.

A canonical multi-variable extension of H^2 is then the Fock space – the Hilbert space of all square-summable power series in d non-commuting variables. In this talk we present several new results on factorization of positive Toeplitz operators on Fock space.

• Distinguished varieties and the Nevanlinna-Pick problem on the symmetrized bidisk

P. Kumar, B.K. Das and H. Sau

AFFILIATION / INSTITUTION: Indian Institute of Science, Bangalore India

EMAIL:poornendukumar@gmail.com, poornenduk@iisc.ac.in Abstract

In this talk, we discuss the uniqueness of the solutions of a solvable Pick interpolation problem in the symmetrized bidisk

$$\mathbb{G} = \{(z_1 + z_2, z_1 z_2) : z_1, z_2 \in \mathbb{D}\}.$$

The uniqueness set is the largest set in $\mathbb{G}$ where all the solutions to a solvable Pick problem coincide. For a solvable Pick problem in $\mathbb{G}$, there is a canonical construction of an algebraic variety, which coincides with the uniqueness set in $\mathbb{G}$. The algebraic variety is called the uniqueness variety. First we discuss that if an N-point solvable Pick problem is such that it has no solutions of supremum norm (over $\mathbb{G}$) less than one and that each of the $(N-1)$-point subproblems has a solution of supremum norm less than one, then the uniqueness variety corresponding to the N-point problem contains a distinguished variety containing all the initial nodes. Here, a distinguished variety is an algebraic variety that intersects the domain $\mathbb{G}$ and exits through its distinguished boundary. Finally, we also discuss about algebraic and geometric characterizations of distinguished varieties.

This talk is based on a joint work with B.K. Das and H. Sau.

References

[1] B.K. Das, P. Kumar and H. Sau, *Distinguished varieties and the Nevanlinna-Pick problem on the symmetrized bidisk.* arXiv:2104.12392 [math.FA]

• Bergman spaces over noncommutative varieties and commutant lifting

Gelu Popescu

University of Texas at San Antonio

We provide analogues of Sarason interpolation theorem in the Hardy algebra $H^\infty(\mathbb{D})$ and Sz.Nagy-Foiaş commutant lifting theorem for contractions on Hilbert spaces in the setting of noncommutative Hardy spaces associated with noncommutative regular domains and varieties. This is accompanied by the study of multi-analytic operators with respect to the universal models associated with the regular domains (resp. varieties) and the study of multipliers of noncommutative Bergman spaces.

As applications, we obtain Toeplitz-corona theorems for multi-analytic operators, commutant lifting in several variables where the liftings are in certain Schur classes, factorization of multi-analytic operators, and Nevanlinna-Pick interpolation results for multipliers of Bergman spaces over Reinhardt domains in $\mathbb{C}^n$.

• Beurling quotient modules on the polydisc

Jaydeb Sarkar, M. Bhattacharjee, B. K. Das, and R. Debnath

Indian Statistical Institute, Bangalore, India

EMAIL: jaydeb@gmail.com

ABSTRACT: Let Q be a closed subspace of the Hardy space over the open unit n-polydisc, $n > 1$. We say that Q is a Beurling quotient module if the orthogonal complement of Q is an inner function-based shift-invariant subspace of the Hardy space. In this talk, along with some applications, we will present a complete classification of Beurling quotient modules. This is joint work with M. Bhattacharjee, B. K. Das, and R. Debnath.

• Realization formulas for noncommutative rational Herglotz-Agler functions

Sanne ter Horst, Estiaan Klem and André Ran

AFFILIATION / INSTITUTION: North West University

EMAIL: sanne.terhorst@nwu.ac.za

Noncommutative (nc) rational matrix functions that are regular at the origin can be realized as transfer functions of nc Givone-Roesser (GR) systems or nc Fornasini-Marchesini (FM) systems, while there are descriptor realisation variations for nc rational matrix functions that are not regular at the origin. However, for specific classes of nc rational matrix functions one would like to have realisation formulas where the system matrices may have certain properties, and for this different type of realisations may have to be considered. For the class of nc rational Herglotz-Agler functions we provide an nc long resolvent type realisation formula analogous to the one that originated in the work of Besmertnyi for the commutative case, and has been studied by several other authors since. We also investigate how such nc long resolvent realisations can be transferred, sometimes under additional conditions, to nc realisations of a different type.

• Noncommutative realizations

Ryan Tully-Doyle and J. E. Pascoe

Cal Poly, San Luis Obispo

rtullydo@calpoly.edu

In the early 20th century, Nevanlinna, Löwner, and Kraus established integral representations that characterize matrix monotone and matrix convex functions in one variable. Structured representations of these types have played a key role in the development of noncommutative function theory, for example the butterfly realization for rational noncommutative functions due to Helton, McCullough, and Vinnikov [1]. We will discuss some recent advances in the theory of realizations for noncommutative functions based on the royal road approach to the noncommutative Löwner theorem [2].

References

[1] J. William Helton, S. McCullough, and V. Vinnikov, *Noncommutative convexity arises from linear matrix inequalities.* J. Funct. Anal. 2006.

[2] J. E. Pascoe and R. Tully-Doyle, *The royal road to automatic noncommutative real analyticity, monotonicity, and convexity.* arXiV preprint. In submission.

4.8 Special Session 8 - Operators in Hypercomplex Analysis

• On the polyanalytic short-time Fourier transform in the quaternionic setting

ANTONINO DE MARTINO, KAMAL DIKI

AFFILIATION / INSTITUTION: POLITECNICO DI MILANO

EMAIL: antonino.demartino@polimi.it ABSTRACT

In this talk we will discuss about a possible extension of the short-time Fourier transform to quaternions. In particular, we consider a short-time Fourier transform, in dimension one, which has the normalized weighted Hermite functions as windows. It turns out that such a transform is connected to the recent theory of slice polyanalytic functions of a quaternionic variable. First of all, we prove some basic results about the Bargmann transform in the polyanalytic framework. Based on the properties of this transform we prove different results on the quaternionic short-time Fourier transform. This talk is based on a joint work with Kamal Diki.

• The Segal-Bargmann transform in Clifford analysis

Swanhild Bernstein and Sandra Schufmann

AFFILIATION / INSTITUTION: TU Bergakademie Freiberg/ Institute of Applied Analysis

EMAIL: swanhild.bernstein@math.tu-freiberg.de

ABSTRACT The Segal-Bargmann transform plays an essential role in signal processing, quantum physics, infinite-dimensional analysis, function theory and further topics. The connection to signal processing is the short-time Fourier transform, which can be used to describe the Segal-Bargmann transform. The classical Segal-Bargmann transform $\mathcal{B}$ maps a square integrable function to a holomorphic function square-integrable with respect to a Gaussian identity. In signal processing terms, a signal from the position space $L_2(\mathbb{R}^m, \mathbb{R})$ is mapped to the phase space of wave functions, or Fock space, $\mathcal{F}^2(\mathbb{C}^m, \mathbb{C})$. We extend the classical Segal-Bargmann transform to a space of Clifford algebra-valued functions. We show how the Segal-Bargmann transform is related to the short-time Fourier transform and use this connection to demonstrate that $\mathcal{B}$ is unitary up to a constant and maps Sommen's orthonormal Clifford Hermite functions $\{\phi_{l,k,j}\}$ to an orthonormal basis of $L^2_\mu(\mathbb{C}^m, \mathcal{C}\ell^{\mathbb{C}}_m)$, $\mu(\underline{z}) = \frac{1}{\pi^m} e^{-|\underline{z}|^2}$.

We also lay out that the Segal-Bargmann transform can be expanded to a convergent series with a dictionary of $L^2_\mu(\mathbb{C}^m, C^{\mathbb{C}}_m)$.

References

S. Bernstein, S. Schufmann *The Segal-Bargmann transform in Clifford analysis.* http://arxiv.org/abs/2106.09956

• Solutions and representation theory of the Lévy-Leblond operator

Hendrik De Bie and Sijia Bao, Denis Constales and Teppo Mertens

Ghent University

Hendrik.DeBie@UGent.be

We determine solutions for the Lévy-Leblond operator or parabolic Dirac operator in terms of hypergeometric functions and spherical harmonics. We subsequently generalise our approach to a wider class of Dirac operators depending on 4 parameters. If time allows, we will also discuss the action of the symmetry algebra on our solutions.

References

[1] S. Bao, D. Constales, H. De Bie, T. Mertens, *Solutions for the Lévy-Leblond or parabolic Dirac equation and its generalizations. J. Math. Phys.* **61**, 011509, 12 pp. (2020)

• Hardy Spaces on a Family of Model Domains in $\mathbf{C}^{n+1}$

Der-Chen Chang

Georgetown University, Washington

chang@georgetown.edu

Consider a family of hypersurfaces in $\mathbf{C}^{n+1}$:

$$\Omega_m = \Big\{(z_1, z_2, \dots, z_n, z_{n+1}) : Im(z_{n+1}) = \big(\sum_{k=1}^{n} |z_k|^2\big)^m\Big\}, \quad m \in \mathbf{N}.$$

In this talk, we establish a Hardy space theory on M_m (the boundary manifold of Ω_m) via a new discrete square function constructed from the heat kernel.

We prove that a class of singular integral operators is bounded on the Hardy spaces $H^p(M_m)$, and are bounded from $H^p(M_m)$ to $L^p(M_m)$ for $\frac{2m+2n}{2m+2n+\vartheta} < p \leq 1$ with $0 < \vartheta < 1$.

As an application, sharp estimates for the fundamental solution of the Kohn Laplacian on $H^p(M_m)$ are derived.

• Poly slice monogenic functions, Cauchy formulas and the PS-functional calculus

KAMAL DIKI, DANIEL ALPAY, FABRIZIO COLOMBO AND IRENE SABADINI

AFFILIATION / INSTITUTION: POLITECNICO DI MILANO

EMAIL: kamal.diki@polimi.it

ABSTRACT

The notion of poly slice analytic functions has been recently introduced in the quaternionic setting. In this talk, we extend this notion to the case of Clifford-valued functions and introduce the poly slice monogenic functions for which we can prove two Cauchy formulas with different kernels. As a consequence, we define the PS-functional calculus associated to these poly slice monogenic functions which is the polyanalytic version of the S-functional calculus. Here also the S-spectrum plays an important role. This talk is based on a joint work with Daniel Alpay, Fabrizio Colombo and Irene Sabadini.

• Symbol calculus of pseudo-differential operators on the group Spin(4)

M. Ferreira[†‡]

[†] School of Technology and Management, Polytechnic of Leiria, P-2411-901, Leiria, Portugal. [‡] CIDMA - Center for Research and Development in Mathematics and Applications. Department of Mathematics, University of Aveiro, Campus Universitário de Santiago, 3810-193 Aveiro, Portugal.

EMAIL: milton.ferreira@ipleiria.pt

In this talk, we consider representation theory to develop a full symbol calculus of pseudo-differential operators on the group Spin(4) in the sense of Ruzhansky-Turunen-Wirth. The essential tools are the Spin(4)-representations, its matrix coefficients, and the Fourier transform on Spin(4), which is a matrix-valued operator. We construct the Spin(4) representations in the spaces of simplicial harmonic polynomials and simplicial spinor-valued monogenic polynomials and decompose them as the tensor product of Spin(3)-representations. Using the Kronecker product and the properties of Spin(3) representations we study recurrence relations for the matrix coefficients of Spin(4) representations and establish a differential and symbol calculus for some left/right invariant differential operators. With the Fourier transform on Spin(4) in hand and a family of admissible first-order difference operators chosen we study pseudo-differential operators on the group Spin(4). We obtain results concerning the ellipticity and the global hypoellipticity of pseudo-differential operators in Spin(4), in terms of their matrix-valued full symbols. Several examples of first and second-order globally hypoelliptic differential operators are given, in particular, of operators that are locally not invertible nor hypoelliptic but globally are.

• On the connection between Fueter's theorem and the generalized CK extension

Alí Guzmán Adán and Kamal Diki and Antonino de Martino

AFFILIATION / INSTITUTION: Ghent University, Belgium

EMAIL: ali.guzmanadan@ugent.be

The Fueter-Sce theorem provides a way of inducing axial monogenic functions in $\mathbb{R}^{m+1}$ from holomorphic intrinsic functions of one complex variable. This result was initially proved for the cases where the dimension m is odd using pointwise differentiation, while the extension to the cases where m is even was proved by Qian using the corresponding Fourier multipliers [3]. These results have been recently unified in the distributional sense [1].

The main goal of this talk is to provide an alternative description of the Fueter-Sce theorem in terms of the generalized axial CK extension. The latter characterizes axial null solutions of the Cauchy-Riemann operator in $\mathbb{R}^{m+1}$ in terms of their restrictions to the real line. This leads to a one-to-one correspondence between the space of axially monogenic functions in $\mathbb{R}^{m+1}$ and the space of analytic functions of one real variable.

We provide explicit expressions for the Fueter-Sce map in terms of the generalized CK extension for both cases, m even and m odd. These expressions allow for a plane wave decomposition of the Fueter map, i.e. a factorization of the Fueter mapping in terms of the dual Radon transform. In turn, this decomposition provides a solution to the problem proposed in [2] related to the extensions of the Coherent State Transform (CST) to Clifford Analysis. In particular, we show how the axial CST defined through the Fueter mapping is related to the axial and slice CST's defined in [2] in terms of the dual Radon transform.

References

[1] B. Dong and T. Qian, *Uniform generalizations of Fueter's theorem.* Ann. Mat. Pura Appl. (4), 200(1):229–251, 2021.

[2] W. D. Kirwin, J. Mourão, J. a. P. Nunes, and T. Qian, *Extending coherent state transforms to Clifford analysis.* J. Math. Phys., 57(10):103505, 10, 2016.

[3] T. Qian, *Generalization of Fueter's result to* $\mathbf{R}^{n+1}$. Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl., 8(2):111–117, 1997.

• The spectral theorem for a normal operator on a Clifford module

David P Kimsey e-mail: david.kimsey@newcastle.ac.uk

In this talk we will consider the problem of obtaining a spectral resolution for a densely defined closed normal operator on a Clifford module $\mathcal{H}_n := \mathcal{H} \otimes \mathbb{R}_n$, where $\mathcal{H}$ is a real Hilbert space and $\mathbb{R}_n := \mathbb{R}_{0,n}$ is the Clifford algebra generated by the units $e_1, \ldots, e_n$ with $e_i e_j = -e_j e_i$ for $i \neq j$ and $e_j^2 = -1$ for $j = 1, \ldots, n$. We shall see that any densely defined closed normal operator T on a Clifford module admits an integral representation which is analogous to the integral representation for a densely defined closed normal operator on a quaternionic Hilbert space (which one may think of as a Clifford module $\mathcal{H}_2$) discovered by Daniel Alpay, Fabrizio Colombo and the speaker in 2014. However, the Clifford module setting sketched above with $n > 2$ presents a number of technical difficulties which are not present in the quaternionic Hilbert space case.

In order to prove this result, one needs to utilise spectra of operators which are not necessarily paravector operators, i.e., operators of the form $T = T_0 + \sum_{j=1}^{n} T_j e_j$. This observation has implications on a generalisation of the S-functional calculus and some related function theory which we shall briefly highlight.

The main thrust of this talk is based on joint work with Fabrizio Colombo. The work on the S-functional calculus is joint work with Fabrizio Colombo, Jonathan Gantner and Irene Sabadini. The work on the related function theory is joint work with Fabrizio Colombo, Irene Sabadini and Stefano Pinton.

• Stochastic PDEs in Clifford analysis: Wick product approach and related results

Swanhild Bernstein and Dmitrii Legatiuk

AFFILIATION / INSTITUTION: Chair of Applied Mathematics, Bauhaus-Universität Weimar, Germany

EMAIL: dmitrii.legatiuk@uni-weimar.de

Although stochastic differential equations play an important role in modelling of real world processes scattering from diffusion and financial markets to describing phenomena in quantum mechanics, their have received only a limited attention in the field of Clifford analysis. Looking at the classical theory of stochastic differential equations, approaches to their analysis can be sub-divided into two general ways: (i) a semi-group approach utilising a semi-group generated by the differential operator of a stochastic DE, and (ii) Wick product approach, where the classical objects of the stochastic calculus, such as e.g. Brownian motion, white noise, and Itô integral, are transferred to the Wick setting. In this talk, we present ideas towards generalising the Wick product approach to the Clifford setting, as well as related results, which might be useful for developing stochastic Clifford analysis, and thus, for studying positive definite functions and reproducing kernels.

References

[1] S. Bernstein, *Towards infinite-dimensional Clifford analysis.* Mathematical Methods in the Applied Sciences, 2021, to appear.

[2] S. Bernstein, D. Legatiuk, *Brownian motion, martingales and Itô formula in Clifford analysis.* Advances in Applied Clifford Algebras, 2021, under review.

• Singularities of bicomplex holomorphic functions

M. Elena Luna–Elizarraras C. Octavio Perez–Regalado and Michael Shapiro

Holon Institute of Technology, Holon, Israel

EMAIL: lunae@hit.ac.il

In the theory of bicomplex holomorphic functions does not exist the concept of isolated singularities; that is, such functions do not have singularities just at a point like holomorphic functions have in one complex variable. However there are other type of singularities that behave similarly to the isolated singularities in one complex variable. In the talk we describe how they can be classified in such a way that it resembles the classification made for the complex analysis case. We will see also that to singularities there corresponds their orders which are hyperbolic numbers with integer components, not real integers. We will mention the Residue Theorem in the bicomplex analysis setting.

• Inverting Radon-type transforms over the Lie sphere using Clifford analysis

Teppo Mertens

AFFILIATION / INSTITUTION: Ghent University

EMAIL: teppo.mertens@ugent.be

In this talk I will consider certain Radon-type transforms over the Lie sphere. More precisely, I will discuss the extended Szegö-Radon, monogenic Hua-Radon and the polarized Hua-Radon transform and how to invert them. These transforms are all dependent on a vector $\underline{\tau} = \underline{t} + i\underline{s}$, where $\underline{t}$ and $\underline{s}$ are perpendicular unit vectors in $\mathbb{R}^m$. Due to the special form of $\underline{\tau}$, we can think of it as a vector in a Stiefel manifold. Taking the average over all these vectors $\underline{\tau}$, we obtain inversion formulas for the aforementioned transforms.

References

[1] D. Constales, H. De Bie, T. Mertens, F. Sommen, *The monogenic Hua-Radon transform and its inverse.* Preprint, arXiv:2011.11457

[2] T. Mertens, F. Sommen, *Inversion for the Hua-Radon and the polarized Hua-Radon transform.* Preprint, arXiv:2102.09279

• The fractional Powers of Quaternionic Vector Operator in Bounded and Unbounded Domains

Stefano Pinton and Fabrizio Colombo, Denis Deniz González

AFFILIATION / INSTITUTION: Politecnico di Milano

EMAIL: stefano.pinton@polimi.it

ABSTRACT Using the spectral theory on the S-spectrum it is possible to define the fractional powers of a large class of vector operators. This possibility leads to new fractional diffusion and evolution problems that are of particular interest for nonhomogeneous materials where the Fourier law is not simply the negative gradient operator but it is a nonconstant coefficients differential operator of the form

$$T = \sum_{\ell=1}^{3} e_\ell a_\ell(x)\partial_{x_\ell}, \quad x = (x_1, x_2, x_3) \in \overline{\Omega},$$

where, Ω can be either a bounded or an unbounded domain in $\mathbb{R}^3$ whose boundary $\partial\Omega$ is considered suitably regular, $\overline{\Omega}$ is the closure of Ω and e_ℓ, for $\ell = 1, 2, 3$ are the imaginary units of the quaternions $\mathbb{H}$. The operators $T_\ell := a_\ell(x)\partial_{x_\ell}$, for $\ell = 1, 2, 3$, are called the components of T and a_1, a_2, $a_3 : \overline{\Omega} \subset \mathbb{R}^3 \to \mathbb{R}$ are the coefficients of T.

In this talk I will discuss the generation of the fractional powers of T, denoted by $P_\alpha(T)$ for $\alpha \in (0, 1)$, when the operators T_ℓ, for $\ell = 1, 2, 3$ do not commute among themselves. To define the fractional powers $P_\alpha(T)$ of T it is sufficient to consider the weak formulation of a suitable boundary value problem associated with the pseudo S-resolvent operator of T. In particular I will explain how to solve this boundary value problem using two different boundary conditions: if Ω is unbounded, I will consider the Dirichlet boundary conditions while, if Ω is bounded, I will consider the natural Robin-type boundary conditions associated with the generation of the fractional powers of T. The last kind of boundary conditions are represented by the operator $\sum_{\ell=1}^{3} a_\ell^2(x)n_\ell(x)\partial_{x_\ell} + a(x)I$, for $x \in \partial\Omega$, where I is the identity operator, $a : \partial\Omega \to \mathbb{R}$ is a given function and $\mathbf{n} = (n_1, n_2, n_3)$ is the outward unit normal vector to $\partial\Omega$. The Robin-type boundary conditions associated with the generation of the fractional powers of T are, in general, different from the Robin boundary conditions associated to the heat diffusion problem which leads to operators of the type $\sum_{\ell=1}^{3} a_\ell(x)n_\ell(x)\partial_{x_\ell} + b(x)I$, $x \in \partial\Omega$. For this reason I will also discuss the conditions on the coefficients a_1, a_2, $a_3 : \overline{\Omega} \subset \mathbb{R}^3 \to \mathbb{R}$ of T and on the coefficient $b : \partial\Omega \to \mathbb{R}$ so that the fractional powers of T are compatible with the physical Robin boundary conditions for the heat equations.

- **Inverse Conductivity Problem a Quaternionic Approach**

Ivan Pombo

ivanpombo@ua.pt

Universidade de Aveiro, Portugal

The inverse conductivity problem was firstly introduced by A.P. Calderón in 1980. The problem consists on the determination of bounded conductivity, with a positive lower bound, inside a body from electrical measurements taken at the boundary, voltage and current, respectively. In two dimensions the problem was solved by Astala and Paivärinta by passing the conductivity equation into a Beltrami equation. To look into the three-dimensional problem we establish a quaternionic-Beltrami equation, based on early work of Santacesaria, and with quaternionic analysis present the foundations for a solution to Calderón problem in three dimensions.

- **Kernel Approximation in Hypercomplex Spaces**

TAO QIAN

AFFILIATION / INSTITUTION: MACAU UNIVERSITY OF SCIENCE AND TECHNOLOGY

EMAIL: fsttq@umac.mo ABSTRACT

A main form of applications of mathematical analysis is approximation. Only for limited cases in one complex variable comprehensive functional approximation theories have been established. For several complex variables and several real variables in the Clifford algebra setting, approximation theory as a general approach has far from bing established. That is partially because the basis theory in relation to uniqueness sets in hypercomplex function spaces is a difficult subject by itself. In the last two decades an adaptive Fourier decomposition theory was developed for one-complex variable that is regardless of the concepts of uniqueness set and basis, while it can achieve effective and practical approximation. The one-complex variable theory is based on Takenaka-Malmquist (T-M) systems. Despite of lack of T-M systems in multi-dimensions the kernel approximation idea may be adapted and generalised to rather general functional spaces in hypercomplex variables and even with matrix-valued functions.

The talk will explain the kernel approximation idea, give a survey on what have recently been achieved, and pose some related open problems.

4.9 Special Session 9 - Operator Theory on reproducing kernel Hilbert spaces

• Berezin regularity of domains in $\mathbb{C}^n$ and the essential norms of Toeplitz operators

Željko Čučković

For the open unit disc $\mathbb{D}$ in the complex plane, it is well known that if $\phi \in C(\overline{\mathbb{D}})$, then its Berezin transform $\widetilde{\phi}$ also belongs to $C(\overline{\mathbb{D}})$. We say that $\mathbb{D}$ is BC-regular. In this paper we study BC-regularity of some pseudoconvex domains in $\mathbb{C}^n$ and show that the boundary geometry plays an important role. We also establish a relationship between the essential norm of an operator in a natural Toeplitz subalgebra and its Berezin transform. (Joint work with Sonmez Sahutoglu)

• Using Representation Theory to Calculate the Spectrum of a Toeplitz Operator

Matthew Dawson (CIMAT), Raúl Quiroga-Barranco (CIMAT), and Gestur Ólafsson (LSU)

CONACYT–CIMAT Unidad Mérida

matthew.dawson@cimat.mx

Toeplitz operators on Bergman spaces of complex-bounded symmetric domains (such as the unit ball $\mathbb{B}^n \subseteq \mathbb{C}^n$ provide an interesting example of a quantization scheme, by associating bounded, real-valued functions on the domain (called "symbols") with self-adjoint operators on the Bergman space. These operators, as is to be expected for a quantization method, do not in general commute, but it was later found that there are several large families of commuting Toeplitz operators on the weighted Bergman spaces of the unit ball $\mathbb{B}^n$ that are associated with symbols invariant under the action of certain subgroups of the group $\mathrm{SU}(n,1)$, which acts by Möbius transformations on the unit ball. The associated unitary actions of the universal covering group $\widetilde{\mathrm{SU}(n,1)}$ on these Bergman spaces are also known in representation theory as scalar-type discrete series representations. Using representation theory it was later possible ([1]) to extend these results, in large part, to the other complex bounded symmetric domains and construct large commuting families of Toeplitz operators.

In this talk we will see how the machinery of representation theory, together with other tools such as Segal-Bargmann transforms, can be used not only to identify commuting families of Toeplitz operators, but also to derive integral formulas for the spectrum of an operator in one of these families ([2,3,4]).

References

[1] M. Dawson, G. Ólafsson, R. Quiroga Barranco. *Commuting Toeplitz operators on bounded symmetric domains and multiplicity-free restrictions of holomorphic discrete series.* J. Funct. Anal. **268** (2015), No. 7, 1711–1732.

[2] Matthew Dawson, Gestur Ólafsson, Raúl Quiroga. *The restriction principle and commuting families of Toeplitz operators on the unit ball.* São Paulo J. of Math. Sci. **12** (2018), No. 2, 196–226.

[3] Matthew Dawson, Gestur Ólafsson, Raúl Quiroga. *Toeplitz operators on the domain* $\{Z \in M_{2\times 2}(\mathbb{Z}) \mid ZZ^* < I\}$ *with* $\mathrm{U}(2) \times \mathbb{T}^2$*-invariant symbols*, en el volumen *Operator Algebras, Toeplitz Operators and Related Topics*, "Operator Theory: Advances and Applications" Vol. 279 (2020), pp. 77–101.

[4] M. Dawson, R. Quiroga Barranco. *Radial Toeplitz operators on the weighted Bergman spaces of Cartan domains.* Representation Theory, Symmetric Spaces, and Integral Geometry (in honor of Gestur Ólafsson's 65th birthday), Contemp. Math. **714** (2018), 97–114.

• On the Boundedness of Toeplitz Operators

Raffael Hagger, Congwen Liu, Jari Taskinen and Jani Virtanen

University of Reading

r.t.hagger@reading.ac.uk

The characterization of bounded Toeplitz operators on the Bergman space of the unit ball is a problem that has been open for many decades. We will have a quick review of the problem and then show some recent results. In particular, a new sufficient condition for boundedness will be derived.

• Compact difference of composition operators on the Hardy spaces

Hyungwoon Koo Department of Mathematics, Korea

University, Seoul 02841, KOREA koohw@korea.ac.kr

In 1981 Berkson [1] found the isolation phenomenon for composition operators acting on $H^2(\mathbf{D})$. Berkson's isolation result was refined later by Shapiro and Sundberg [8] and they raised the question of *whether two composition operators form a compact difference if they belong to the same path component*. Later their question was answered negatively; see [2], [5] and [7]. On the other hand, by such a negative result, the problem of characterizing compact differences of composition operators became more interesting. The compact difference on the Bergman spaces has been characterized in 2005([6]) but it had been open until quite recently for Hardy space. In this talk we present a measure theoretic characterization of the compact difference of composition operators on the Hardy space.

References

[1] E. Berkson, *Composition operators isolated in the uniform operator topology*, Proc. Amer. Math. Soc., **81**(1981), 230–232.

[2] P. Bourdon, *Components of linear fractional composition operators*, J. Math. Anal. Appl. **279**(2003), 228–245.

[3] B. R. Choe, K. Choi, H. Koo and J. Yang, *Difference of weighted composition operators*, J. Funct. Anal. **278**(2020), Article: 108401.

[4] B. R. Choe, K. Choi, H. Koo and I. Park, *Compact difference of composition operators on the Hardy spaces, preprint.*

[5] E. Gallardo-Gutiérrez, M. González, P. Nieminen and E. Saksman, *On the connected component of compact composition operators on the Hardy space*, Adv. Math. **219**(2008), 986–1001.

[6] J. Moorhouse, *Compact differences of composition operators*, J. Funct. Anal. **219**(2005), 70–92.

[7] J. Moorhouse and C. Toews, *Differences of composition operators: Trends in Banach Spaces and Operator Theory*, Contemp. Math. **321**(2003), 207–213.

[8] J. Shapiro and C. Sundberg, *Isolation amongst the composition operators*, Pacific J. Math. **145**(1990), 117–152.

• p-Summable commutators in Bergman spaces on egg domains

Mohammad Jabbari

AFFILIATION / INSTITUTION: Centro de Investigacion en Matematicas (CIMAT)

EMAIL: mohammad.jabbari@cimat.mx

ABSTRACT: The exact range of the positive real parameter p is determined so that the commutators in the C*-algebra of Toeplitz operators associated to continuous symbols and acting on Bergman spaces over generalized complex ellipsoids are Schatten p-summable.

• Homogeneously polyanalytic kernels on the unit ball and the Siegel domain

<u>Christian Rene Leal-Pacheco</u>, Egor Maximenko and Gerardo Ramos-Vazquez

Affiliation: National Polytechnic Institute of Mexico

email: christian.leal.pacheco@gmail.com

We prove that the homgeneously polyanalytic functions of order m defined by the system of equations given by the Wirtinger derivative $\overline{D}^{(k_1,\ldots,k_n)} f = 0$ with $k_1 + \cdots + k_n = m$ can be written as polynomials of degree $< m$ in variables $\overline{z_1}, \ldots, \overline{z_n}$, with some analytic coefficients. We establish a weighted mean value property of Jacobi polynomials. After that, we give a general recipe to transform a reproducing kernel by a weighted change of variables. Applying these tools, we compute the reproducing kernel of the Bergman space of the homogeneously polyanalytic functions on the unit ball in $\mathbb{C}^n$ and on the Siegel domain. For the one-dimensional case, analogous results were obtained by Koshelev (1997), Pessoa (2014), Hachadi and Youssfi (2019).

References

[1] Leal-Pacheco, C.R.; Maximenko, E.A.; Ramos-Vazquez, G. (2021): *Homogeneously polyanalytic kernels on the unit ball and the Siegel domain.* Complex Anal. Oper. Th. (to appear).

• Radial operators on polyanalytic weighted Bergman spaces

Roberto Moisés Barrera-Castelán, Egor Maximenko, and Gerardo Ramos Vazquez

Affiliation: National Polytechnic Institute of Mexico

email: emaximenko@ipn.mx

Let $\mathcal{A}_n^2$ be the space of the n-analytic functions on the unit disk $\mathbb{D}$, square-integrable with respect to the weighted Lebesgue measure $\frac{\alpha+1}{\pi}(1-|z|^2)^\alpha\,\mathrm{d}\mu$. Extending results of Ramazanov (1999, 2002), we explain that disk polynomials (studied by Koornwinder in 1975 and Wünsche in 2005) form an orthonormal basis of $\mathcal{A}_n^2$. Using this basis, we provide the Fourier decomposition of $\mathcal{A}_n^2$ into the orthogonal sum of the subspaces associated to different frequencies. This leads to the decomposition of the von Neumann algebra of radial operators, acting in $\mathcal{A}_n^2$, into the direct sum of some matrix algebras. In other words, all radial operators are represented as matrix sequences. In particular, we represent in this form the Toeplitz operators with bounded radial symbols, acting in $\mathcal{A}_n^2$. Moreover, using ideas by Engliš (1996), we show that the set of the Toeplitz operators with bounded generating symbols is not weakly dense in $\mathcal{B}(\mathcal{A}_n^2)$.

References

[1] Barrera-Castelán, R.M.; Maximenko, E.A.; Ramos-Vazquez, G. (2021): *Radial operators on polyanalytic weighted Bergman spaces.* Bol. Soc. Mat. Mex. 27, 43.

• TOEPLITZ OPERATORS WITH ISOTROPIC INVARIANT SYMBOLS ON THE FOCK SPACE

ARMANDO SÁNCHEZ-NUNGARAY

AFFILIATION / INSTITUTION: UNIVERSIDAD VERECRUZANA

EMAIL: armsanchez@uv.mx ABSTRACT

We define the so-called k-horizontal and k-cohorizontal sym- bols and study the C^*-algebras generated by Toeplitz operators with these symbols. We show that the C^*-algebra generated by Toeplitz operators which commute with every Toeplitz operator with k-cohorizontal symbols coincides with the C^*-algebra generated by Toeplitz operators with k-horizontal symbols.

Moreover, we prove that this result is also true for Toeplitz operators whit symbols invariant under translations over any isotropic subspace.

That is, the C^*-algebra generated by Toeplitz operators which commute with every Toeplitz operator with coisotropic invariant symbols coincides with the C^*-algebra generated by Toeplitz operators whose symbols are invariant under translations of the corresponding isotropic subspace.

• C^*-algebras generated by quasi-invariant symbols acting on the Fock space on $\mathbb{C}^n$

SPEAKER: Gestur Ólafsson and Vishwa N. Dewage

AFFILIATION / INSTITUTION: Louisiana State University

EMAIL: olafsson@math.lsu.edu ABSTRACT

Let $\mathcal{F}_n$ be the Fock space of holomorphic functions on $\mathbb{C}^n$ which are square integrable with respect to the measure $d\mu_n(z) = \pi^{-n}e^{-|z|^2}dz$. We will discuss some classes of Toeplitz operators that are generated by symbols that are invariant under the action of certain subgroups of the unitary group U_n. We show that those are C^*-algebras. Furthermore we give an integral formula for the symbol of those operators, a formula that can then be used to describe the C^*-algebra in more details.

• Toeplitz operators on the unit ball and moment maps of Abelian groups

Raul Quiroga-Barranco

AFFILIATION / INSTITUTION: Centro de Investigacion en Matematicas, Guanajuato, Mexico

EMAIL:quiroga@cimat.mx

Toeplitz operators with special symbols have provided interesting C^*-algebras. In particular, Vasilevski and the author have proved that, on the unit ball $\mathbb{B}^n$, for every maximal Abelian subgroup (MASG) G of its automorphism group $\mathrm{SU}(n, 1)$, the G-invariant essentially bounded symbols yield Toeplitz operators generating a commutative C^*-algebra.

The unit ball $\mathbb{B}^n$ is well known to be a Kähler manifold carrying a natural symplectic structure invariant under $\mathrm{SU}(n, 1)$. This allows to consider the so-called moment map for a MASG G of $\mathrm{SU}(n, 1)$. Sanchez-Nungaray

and the author computed the moment map of such groups G, and their subgroups, and used this to reveal some interesting relationship between symplectic geometry and Toeplitz operators on $\mathbb{B}^n$.

In this talk we will discuss this application of symplectic geometry and moment map theory. Among other results, we will describe a construction that shows how to obtain, from any Abelian subgroup of $\mathrm{SU}(n, 1)$, a commutative C^*-algebra generated by Toeplitz operators acting on $\mathbb{B}^n$.

This is joint work with Armando Sanchez-Nungaray.

References

[1] Quiroga-Barranco, Raul; Sanchez-Nungaray, Armando, *Moment maps of Abelian groups and commuting Toeplitz operators acting on the unit ball.* Journal of Functional Analysis 281 (2021), no. 3, article 109039.

[1] Quiroga-Barranco, Raul; Vasilevski, Nikolai, *Commutative C^*-algebras of Toeplitz operators on the unit ball. I. Bargmann-type transforms and spectral representations of Toeplitz operators..* Integral Equations and Operator Theory 59 (2007), no. 3, 379-419.

- **Recent results on Bergman kernel estimates and Toeplitz operators in weighted Bergman-type spaces.**

Jari Taskinen, University of Helsinki

We review recent results on the boundedness of Bergman projections and on the Bergman kernel estimates in weighted Bergman spaces A^p_v, $1 < p < \infty$, or H^∞_v, of the unit disc, where the weights are rapidly decreasing and the spaces are "large". We use a number of techniques based e.g. on estimates of the Taylor series. We also review some progress in the questions of boundedness, compactness, Fredholm and spectral properties of Toeplitz operators in Bergman spaces of several variables.

The results are contained in a series of papers published together with José Bonet, Raffael Hägger, Congwen Liu, Wolfgang Lusky and Jani Virtanen.

- **When does $T_f T_g - T_h$ have finite rank?**

Trieu Le and Damith Thilakarathna

University of Toledo

trieu.le2@utoledo.edu

If T_f and T_g are two bounded Toeplitz operators on the Bergman space over the unit disk, then the product $T_f T_g$ is not a Toeplitz operator in general. Researchers have been interested in determining conditions on the symbols f and g for which $T_f T_g$ is a Toeplitz operator or a finite rank perturbation of a Toeplitz operator. Even though the general problem remains wide open, progresses have been made under various restrictions. In recent joint work with D. Thilakarathna, we discover a new noncommutative convolution and use it to obtain a complete solution when f and g are linear combinations of certain quasihomogeneous functions. Our approach applies to finite sums of Toeplitz products as well.

- ## Fredholm properties of Toeplitz operators on doubling Fock spaces

Jani Virtanen

AFFILIATION / INSTITUTION: University of Reading, UK.

EMAIL:j.a.virtanen@reading.ac.uk

The Fredholm properties of a large class of Toeplitz operators on weighted Fock spaces are well understood when the Laplacian of the weight function is bounded below and above in the complex plane. In particular, this includes the standard weights and Fock-Sobolev weights. In this talk we extend the characterization to doubling Fock spaces with a subharmonic weight whose Laplacian is a doubling measure. The main difficulty is the more complicated geometry induced by the Bergman metric for doubling Fock spaces. Joint work with Zhangjian Hu.

- ## Localization of Toeplitz operators with BMO symbols

Nina Zorboska

University of Manitoba

Nina.Zorboska@umanitoba.ca ABSTRACT

Toeplitz operators with BMO symbols on the Bergman space provide a class of Toeplitz operators with possibly unbounded symbols for which their boundedness and compactness can be determined by their Berezin transform. Even though there was a direct proof of this provided years

a go, the recent research on localization properties of operators on the Bergman space provides yet another view on why that happens with this particular class of operators.

4.10 Special Session 10 - Pseudo-differential Operators

• Bilinear pseudo-differential operators with Gevrey-Hörmander symbols

Ahmed Abdeljawad Johann Radon Institute for Computational and Applied Mathematics, Austrian Academy of Sciences, Linz, Austria

ahmed.abdeljawad@ricam.oeaw.ac.at

We consider bilinear pseudo-differential operators whose symbols may have a sub-exponential growth at infinity, together with all their derivatives. It is proved that those symbol classes can be described by the means of the short-time Fourier transform and modulation spaces. Our first main result is the invariance property of the corresponding bilinear operators. Furthermore we prove the continuity of such operators when acting on modulation spaces. As a consequence, we derive their continuity on anisotropic Gelfand-Shilov type spaces.

The talk is based on joint work with Prof. Sandro Coriasco and Prof. Nenad Teofanov cf. [1].

References

[1] A. Abdeljawad, S. Coriasco and N. Teofanov. Bilinear Pseudo-differential Operators with Gevrey–Hörmander Symbols. Mediterr. J. Math. 17, 120 (2020). https://doi.org/10.1007/s00009-020-01546-y

• Weighted Orlicz Amalgam Spaces on Locally Compact Groups

B. ARIS and S. ÖZTOP

AFFILIATION / INSTITUTION: Department of Mathematics / Istanbul University

EMAIL: busra.unal@istanbul.edu.tr

Let G be a locally compact group, $\Phi_1,\ \Phi_2$ be Young functions and ω be a weight function on G. We study the weighted Orlicz amalgam spaces

$W(L^{\Phi_1}(G),\ L^{\Phi_2}_\omega(G))$ where the local component space is $L^{\Phi_1}(G)$ and the global component is $L^{\Phi_2}_\omega(G)$. We derive some basic properties of these spaces, such as translation invariance, duality and inclusion relations. We also obtain an equivalent discrete norm on $W(L^{\Phi_1}(G),\ L^{\Phi_2}_\omega(G))$, and by using it we obtain some convolution theorems.

References

[1] Feichtinger, H. G., *Banach Convolutions Algebras of Wiener's Type.* Colloq. Math. Soc. Janos Bolyai (1980).

[2] Heil, C., *An Introduction to Weighted Wiener Amalgams.* R. Ramakrishnan and S. Thangavelu, eds., Proc. International Cconference on Wavelets and their Applications (Chennai, 2002), Allied Publishers, New Delhi (2003).

[3] Rao, M. M., Ren, Z. D., *Theory of Orlicz Spaces.* CRC Press.

• Globally hypoelliptic time-periodic evolution equations

Fernando de Ávila Silva

Department of Mathematics / Federal University of Paraná, Brazil.

fernando.avila@ufpr.br

In this talk, we present results on the investigation of the global hypoellipticity problem for operators of type

$$L \doteq D_t + C(t, x, D_x),\ D_t = i^{-1}\partial_t,\ t \in \mathbb{T},\ x \in M,$$

where $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$ is the unit circle, and $C(t, x, D_x)$ is a (pseudo)differential operator on M, smoothly depending on the periodic variable t. If M is a compact manifold, we investigate the C^∞-hypoellipticity, namely, the global regularity for solutions of the problem

$$Lu \in C^\infty(\mathbb{T} \times M),\ u \in \mathcal{D}'(\mathbb{T} \times M),$$

while in case $M = \mathbb{R}^n$ we consider the setting

$$Lu \in \mathscr{F}_\mu(\mathbb{T} \times \mathbb{R}^n),\ u \in \mathscr{U}_\mu(\mathbb{T} \times \mathbb{R}^n),\ \mu \geq 1/2,$$

where $\mathscr{F}_\mu$ and $\mathscr{U}_\mu$ denotes classes of *time-periodic Gelfand-Shilov spaces.*

The key tool in our approach is a characterization of the functional spaces in view of a Fourier analysis given by eigenfunction expansions generated by a fixed elliptic (pseudo)differential operator on M.

Parts of the talk are based on joint works with M. Cappiello, T. Gramchev and A. Kirilov.

• Boundedness of maximal singular operators via the LGC-method

Árpád Benyi

Department of Mathematics, Western Washington University

benyia@wwu.edu

We discuss the LGC-method introduced by V. Lie to study large classes of sub-bilinear maximal operators with highly singular symbols. In particular, we investigate the continuity of operators stemming from a maximally modulated bilinear Hilbert transform along curves. This is joint work with V. Lie (Purdue University).

• Wigner Analysis of Operators

Elena Cordero and Luigi Rodino

Dipartimento di Matematica, University of Torino, Italy

elena.cordero@unito.it

We perform a τ-Wigner analysis of linear operators. The time-frequency representation *Short-time Fourier Transform* (STFT) is replaced by τ-Wigner distributions. Such representations provide a new characterization for modulation spaces when $\tau \in (0,1)$. We show that they can be efficiently employed in the study of the off-diagonal decay for pseudodifferential operators with symbols in the Sjöstrand class (in particular, in the Hörmander class $S^0_{0,0}$). We deduce micro-local properties for pseudodifferential operators in terms of the Wigner wave-front set. The major advantage of the Wigner kernel versus the STFT one resides in study of the Schrödinger equation with quadratic Hamiltonians.

• Estimates for time-dependent multipliers with oscillatory and diffusive components

Marcelo Rempel Ebert and Marcello D'Abbicco

AFFILIATION / INSTITUTION: University of São Paulo

EMAIL: ebert@ffclrp.usp.br ABSTRACT

In this talk, we derive long time $L^p - L^q$ decay estimates, in the full range $1 \leq p \leq q \leq \infty$, for time-dependent multipliers in which an interplay between an oscillatory component and a diffusive component with different scaling appears. We estimate $\|m(t,\cdot)\|_{M_p^q}$ (see [1]) as $t \to \infty$ for multipliers of type

$$m(t,\xi) = e^{\pm\, i|\xi|^\sigma t - |\xi|^\theta t},$$

and suitable perturbations, under the assumption that at low frequencies the scaling of the diffusive component is worse, i.e., $\theta \in (\sigma, 2\sigma]$. These multipliers are, for instance, related to the fundamental solution to the Cauchy problem for the viscoelastic plate equation:

$$u_{tt} + \Delta^2 u + \Delta^2 u_t = 0, \quad t \geq 0,\ x \in \mathbb{R}^n.$$

References

[1] L. Hörmander, *Estimates for translation invariant operators in L^p spaces. Acta Mathematica* **104** (1960), 93–140.

• Fourier multipliers, the mixed frame operator and reproducing formulas for generalized shift-invariant systems

Anupam Gumber

Indian Institute of Science Bangalore, India

EMAIL: anupamgumber@iisc.ac.in

For a given pair of frames $\{\psi_n\}$ and $\{\varphi_n\}$ in a separable Hilbert space $\mathcal{H}$, the associated mixed frame operator $S : \mathcal{H} \to \mathcal{H}; f \mapsto \sum_n \langle f, \psi_n\rangle \varphi_n$ is a bounded linear operator. In this talk we will present a characterization result for the mixed frame operator to be a Fourier multiplier which concerns a concept at the core of frame theory, namely, the reproducing formulas for frame pairs $\{\psi_n\}$ and $\{\varphi_n\}$ in $\mathcal{H}$. The result turned out to be not only

interesting in itself, but also important for further investigations. We will present some properties of S and apply the obtained characterization to investigate reproducing (reconstruction) property when $\{\psi_n\}$ and $\{\varphi_n\}$ belong to generalized shift-invariant systems, a special class of structured frame systems motivated by the utility of a recent notion considered in [1] and [2].

References

[1] J. W. Iverson, *Subspaces of $L^2(G)$ invariant under translation by an abelian subgroup.* J. Funct. Anal., 269(3) (2015), 865-913.

[2] M. S. Jakobsen and J. Lemvig, *Reproducing formulas for generalized translation invariant systems on locally compact abelian groups.* Trans. Amer. Math. Soc., 368 (2016), 8447-8480.

• Boundedness of FIOs with Amplitudes in General Hörmander Classes

Anders Israelsson

AFFILIATION / INSTITUTION: Uppsala University

EMAIL: anders.israelsson@math.uu.se

Fourier integral operators (FIOs) are a kind of oscillatory integral operators that are used to solve hyperbolic partial differential equations. In 1991 Andreas Seeger, Christopher D. Sogge and Elias M. Stein proved the sharp local L^p-boundedness for FIOs with amplitudes in the Hörmander classes $S^m_{\rho,1-\rho}(\mathbb{R}^n)$, $1/2 \leq \rho \leq 1$. In a recent joint work with Alejandro J. Castro and Wulf Staubach we have obtained a significant improvement of this result, in the sense that we establish the global boundedness of FIOs with amplitudes in $S^m_{\rho,\delta}(\mathbb{R}^n)$, with $0 \leq \rho \leq 1$, $0 \leq \delta < 1$. In the talk we discuss this result and a sketch of the proof.

References

[1] A.J. Castro, A. Israelsson, and W. Staubach, *Regularity of Fourier integral operators with amplitudes in general Hörmander classes*, to appear in Analysis and Mathematical Physics. (arXiv: 2003.12878)

[2] A. Seeger, C. D. Sogge, and E. M. Stein, *Regularity properties of Fourier integraloperators*, Ann. of Math. (2), 134 (1991), pp. 231–251.

• MELLIN PSEUDODIFFERENTIAL OPERATORS WITH QUASICONTINUOUS SYMBOLS AND THEIR APPLICATIONS

YURI KARLOVICH

INSTITUTION: Universidad Autónoma del Estado de Morelos, México

EMAIL: karlovich@uaem.mx

Mellin pseudodifferential operators with quasicontinuous $V(\mathbb{R})$-valued symbols on the Lebesgue spaces $L^p(\mathbb{R}_+, d\mu)$ with $p \in (1, \infty)$ are studied, where $V(\mathbb{R})$ is the Banach algebra of absolutely continuous functions of bounded total variation on the real line $\mathbb{R}$, and $d\mu$ is an invariant measure on $\mathbb{R}_+$. Applying obtained results on Mellin pseudodifferential operators with non-regular symbols, we study a Banach algebra $\mathfrak{B}_p$ generated by singular integral operators with quasicontinuous data on the space $L^p(\mathbb{R}_+)$ and by products of shift operators and singular integral operator with point singularities at 0 and ∞. A Fredholm symbol calculus for the Banach algebra $\mathfrak{B}_p$ is constructed and a Fredholm criterion for the operators $A \in \mathfrak{B}_p$ is established.

• On Fourier integral operators and maximal operators

Ramesh Manna

AFFILIATION / INSTITUTION: National Institute of Science Education and Research (NISER) Bhubaneswar, India

EMAIL: rameshmanna@niser.ac.in

Abstracts The theory of Fourier integral operators was developed by Hörmander in 1971. In this talk, we discuss the local smoothing estimates for Fourier integral operators with phase function $h(x, t, \xi) = x.\xi + tq(\xi)$, where q is smooth, homogeneous of degree one and amplitude function $a(x, t, \xi)$ belongs to S^m, the symbol class of order m less or equal to zero. Local smoothing was a phenomenon originally observed in studying the circular maximal operator by C. D. Sogge. We give an overview of the regularity results which have been proven to date. Finally, we give an application of the local smoothing estimate to the maximal operators. This is a joint work with Prof. P. K. Ratnakumar.

References

[1] Ramesh Manna; P. K. Ratnakumar, *Local smoothing of Fourier integral operators and Hermite functions.* Trends Math., 2020

[2] Ramesh Manna, *Maximal functions associated to flat plane curves with Mitigating factors.* Annali di Matematica Pura ed Applicata (1923 -), 2019.

- **The Hartree-Fock equations in modulation spaces**

Kasso Okoudjou

Abstract: In this talk, we establish both a local and a global well-posedness theories for the nonlinear Hartree-Fock equations and its reduced analog in the setting of the modulation spaces on $\mathbb{R}^d$. In the process, we prove the boundedeness of certain multilinear operators on products of the modulation spaces.

This talk is based on joint work with D. Bhimani and M. Grillakis

- **Invariance of the Fredholm Index of Non-Smooth Pseudodifferential Operators**

Dr. Christine Pfeuffer

As nearly invertible operators Fredholm operators play an important role in the field of partial differential equations in order to obtain existence and uniqueness results. Hence great effort already was spent to get some conditions for the Fredholmness of pseudodifferential operators. However, there are very few results for the invariance of the Fredholm index of such operators.

In the smooth case Schrohe was able to show under certain conditions, that the Fredholm index of smooth pseudodifferential operators is invariant considered as a map between certain weighted Bessel potential spaces with symbols in the Hörmander-class $S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$.

In applications also non-smooth pseudodifferential operators occur. The goal of this talk is to show the invariance of the Fredholm index for non-smooth pseudodifferential operators with symbols in the class $C^{\tilde{m},s}S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$. To reach this aim we use the main idea of the result from Rabier about the Fredholm index for non-smooth differential operators. The main difficulty is to prove a regularity result for non-smooth pseudodifferential operators needed in the proof.

The talk is based on a joint work with H. Abels.

• Hardy spaces for generalised Fourier Integral Operators

Pierre Portal and Dorothee Frey, Andrew Hassell, Jan Rozendaal

Australian National University

pierre.portal@anu.edu.au

The (local version) of the classical Hardy space $H^1(\mathbb{R}^d)$ can be seen as the largest subspace of $L^1(\mathbb{R}^d)$ invariant under the action of pseudo-differential operators of order 0. In [1], Hart Smith introduced a new Hardy space H^1_{FIO}, that is invariant under the action of Fourier Integral Operators of order 0. His space is built over phase space, and is reminiscent of modulation spaces. In this talk, I discuss recent generalisations of Smith's construction. This includes H^p_{FIO} spaces which embed optimally into the scale of Sobolev spaces for the wave equation (i.e. loosing $(d-1)|\frac{1}{p}-\frac{1}{2}|$ derivatives), whereas modulation spaces embed optimally for the Schrödinger equation (i.e. loosing $2d|\frac{1}{p}-\frac{1}{2}|$ derivatives). In the past 15 years, standard Hardy spaces have also been generalised by replacing the convolution operators used in their definitions by solution operators arising from a given heat equation. I will also discuss how this philosophy can be used with H^p_{FIO} spaces, by adapting wave packet transforms to rough Dirac operators (instead of the usual partial derivatives). This gives fixed time L^p estimates with optimal loss of derivatives for certain wave equations with Lipschitz coefficients.

References

[1] Hart Smith, *A Hardy space for Fourier integral operators.* J. Geom. Anal., 8(4) (1998) 629–653.

[2] Andrew Hassell, Pierre Portal, Jan Rozendaal, *Off singularity bounds and Hardy spaces for Fourier integral operators.* Transactions of the American Mathematical Society 373 (2020) 5773–5832.

[3] Dorothee Frey, Pierre Portal, *L^p estimates for wave equations with specific Lipschitz coefficients.* arXiv:2010.08326.

• Characterisation of the Weyl-Hörmander Classes Via Growth Estimates of Time-Frequency Shifts

Stevan Pilipović and Bojan Prangoski

AFFILIATION / INSTITUTION: Faculty of Mechanical Engineering, Ss. Cyril and Methodius University in Skopje, Macedonia

EMAIL: bprangoski@yahoo.com ABSTRACT

Inspired by the characterisation of the Sjöstrand symbol classes in [2], Gröchenig and Rzeszotnik [1] gave the following characterisation of the elements of the Hörmander class $S^0_{0,0}$. A tempered distribution $a \in \mathcal{S}'(\mathbb{R}^{2n})$ belongs to $S^0_{0,0}$ if and only if for every $s > 0$ there is $C_s > 0$ such that

$$|\langle a^w \pi(X)\chi, \overline{\pi(\Xi)\chi}\rangle| \leq C_s(1 + |X - \Xi|)^{-s}, \quad \text{for all}\, X, \Xi \in \mathbb{R}^{2n};$$

here $\chi \in \mathcal{S}(\mathbb{R}^n)\backslash\{0\}$ and $\pi(X)\chi(y) = e^{2\pi i\xi y}\chi(y-x)$ for $X = (x, \xi) \in \mathbb{R}^{2n}$. In this talk, we present a generalisation of this result which characterises the elements of the Weyl-Hörmander symbol classes $S(M, g)$ with g a Hörmander metric on $\mathbb{R}^{2n}$ and M a g-admissible weight. When g is the Euclidean metric and $M = 1$, $S(M, g)$ is just $S^0_{0,0}$ and in this case our result reduces to the above mentioned result of Gröchenig and Rzeszotnik [1].

References

[1] K. Gröchenig, Z. Rzeszotnik, *Banach algebras of pseudodifferential operators and their almost diagonalization*, Annales de l'institut Fourier 58(7) (2008), 2279-2314.

[2] J. Sjöstrand, *Wiener type algebras of pseudodifferential operators*, Séminaire Équations aux dérivées partielles (Polytechnique) (1995), 1-19.

• Uncertainty principles and null-controllability of evolution equations enjoying Gelfand-Shilov smoothing effects

KAREL PRAVDA-STAROV and JÉRÉMY MARTIN

AFFILIATION / INSTITUTION: Université de Rennes 1

EMAIL: karel.pravda-starov@univ-rennes1.fr

ABSTRACT : We discuss uncertainty principles for finite combinations of Hermite functions and establish some spectral inequalities for control

subsets that are thick with respect to some unbounded densities growing almost linearly at infinity. These spectral inequalities allow to derive the null-controllability in any positive time for evolution equations enjoying specific regularizing effects in Gelfand-Shilov spaces. This is a joint work with Jérémy Martin (Université de Rennes 1)

References

[1] J. Martin, K. Pravda-Starov, *Spectral inequalities for combinations of Hermite functions and null-controllability for evolution equations enjoying Gelfand-Shilov smoothing effects.* submitted for publication

• Parametric pseudodifferential operators with point-singularity in the covariable

JÖRG SEILER

AFFILIATION / INSTITUTION: Università degli Studi di Torino

EMAIL: joerg.seiler@unito.it

Starting out from a new description of a class of parameter-dependent pseudodifferential operators with finite regularity number due to G. Grubb, we introduce a calculus of parameter-dependent, poly-homogeneous symbols whose homogeneous components have a particular type of singularity in the covariable-parameter space. Such symbols admit intrinsically a second kind of expansion which is closely related to the expansion in the Grubb-Seeley calculus and permits to recover the resolvent-trace expansion for elliptic pseudodifferential operators originally proved by Grubb-Seeley. Another application is the invertibility of parameter-dependent operators of Toeplitz type, i.e., operators acting in subspaces determined by zero-order pseudodifferential idempotents.

References

[1] J. SEILER, *PARAMETRIC PSEUDODIFFERENTIAL OPERATORS WITH POINT-SINGULARITY IN THE COVARIABLE.* PREPRINT, arxiv:1812.07251

• Disjoint dynamics on weighted Orlicz spaces

Serap Öztop Kaptanoğlu

Department of Mathematics, Faculty of Science, İstanbul University, İstanbul, Turkey oztops@istanbul.edu.tr

Let G be a locally compact group, ω be a weight on G and Φ be a Young function. We give some characterizations for translation operators on the weighted Orlicz space $L^{\Phi}_{\omega}(G)$ to be disjoint topologically transitive and disjoint topologically mixing. In particular, we show that, in certain cases, operators are disjoint topologically transitive if and only if their direct sum is topologically transitive.

This is joint work with Chung-Chuan Chen and Seyyed Mohammad Tabatabaie.

• Pseudodifferential calculi using Weyl pairs in L^p

Himani Sharma

Australian National University

himani.sharma@anu.edu.au

A pseudo-differential calculus on $\mathbb{R}^d$ can be seen as a joint functional calculus of standard position and momentum operators, $Q = (Q_1, ..., Q_d)$ and $P = (P_1, ..., P_d)$, given as multiplication by x_j and partial derivatives ∂_{x_j}, respectively. This calculus is defined for $a \in \mathcal{S}(\mathbb{R}^d)$ by

$$a(Q,P)f := \frac{1}{(2\pi)^d} \int_{\mathbb{R}^{2d}} \hat{a}(u,v) e^{iuQ+ivP} f dudv; \; f \in L^2(\mathbb{R}^d)$$

It can be generalised to a calculus of Weyl pairs acting on a Banach space X. A pair (A, B) of d-tuples $A = (A_1, ..., A_d)$ and $B = (B_1, ..., B_d)$ is called a Weyl pair if $iA_1, ..., iA_d$ and $iB_1, ..., iB_d$ generate bounded C_0-groups on X satisfying the following canonical commutation relation:

$$e^{isA_j} e^{itA_k} = e^{itA_k} e^{isA_j}; \;\; e^{isB_j} e^{itB_k} = e^{itB_k} e^{isB_j}$$
$$e^{isA_j} e^{itB_k} = e^{-ist\delta_{jk}} e^{itB_k} e^{isA_j}$$

In this talk, we will show spectral multiplier estimates on this calculus for Weyl pairs and the abstract Harmonic oscillator obtained using the sum of the squares of these pairs. This generalises standard pseudo-differential operator estimates to abstract functional calculi with similar algebraic properties.

References

[1] C. Kriegler and L. Weis. *Spectral multiplier theorems via H^∞-calculus and R-bounds.* Math. Z., 289(1-2):405-444, 2018.

[2] P.C. Kunstmann and A.Ullmann. *$\mathcal{R}_s$-bounded H^∞-calculus for sectorial operators via generalized Gaussian estimates.* Math. Nachr., 288(11-12):1371-1387,2015

[3] J. van Neerven and P. Portal. *The Weyl calculus for group generators satisfying the canonical commutation relation.* J. Operator Theory, 83(2):253-298,2020.

• Analytic pseudodifferential operators, some classical and recent considerations

N. Teofanov and J. Toft, P. Wahlberg

University of Novi Sad, Novi Sad, Serbia

nenad.teofanov@dmi.uns.ac.rs

The first part of the lecture is devoted to historical background and motivation for our research. 95 years ago (1926), Schrödinger published a series of papers devoted to his approach to quantum mechanics. Soon after (1928), Vladimir Fock proposed a correspondence related to canonical commutation relations of Schrödinger's approach. This includes operators acting on a space of analytic functions, nowadays called the Fock (or Bargmann) space. Back in 1961, Valentine Bargmann published a paper which gave a solid mathematical foundation to the Fock correspondence. This was further elaborated in Bargmann's paper from 1967 in the framework of tempered distributions.

Recently, Joachim Toft studied the mapping properties of the Bargmann transform when acting on different families of test functions and their distribution spaces. We will give a brief outline of those results, emphasizing the role of Hermite functions and linear harmonic oscillator in such investigations.

We proceed with a selection of results from [1] and [2] related to analytic pseudodifferential operators. In particular, we consider Wick and anti-Wick connection, and comment how our approach can be used to recover and improve some known results in the context of real analysis.

References

[1] Nenad Teofanov, Joachim Toft, *Pseudo-differential calculus in a Bargmann setting.* Annales Academiae Scientiarum Fennicae Mathematica, 45 (1), 2020, 227-257.

[2] Nenad Teofanov, Joachim Toft, Patrik Wahlberg, *Pseudo-differential operators with isotropic symbols, Wick and anti-Wick operators, and hypoellipticity.* arXiv:2011.00313

• GABOR ANALYSIS OF QUASI ORLICZ MODULATION SPACES

Rüya Üster

AFFILIATION / INSTITUTION: İstanbul University, İstanbul, Turkey

EMAIL:ruya.uster@istanbul.edu.tr ABSTRACT

Let $(\Omega_j, \Sigma_j, \mu_j)$ be Borel measure spaces with $\Omega_j \subseteq \mathbf{R}^{d_j}$, $\Phi_{0,j}$ be Young functions, Φ_j be quasi-Young functios of order $r_0 \in (0,1]$ given by $\Phi_j(t) = \Phi_{0,j}(t^{r_0})$, $t \geq 0$ and let ω be a moderate weight on $\mathbf{R}^{d_1+d_2}$. In this talk we will extend the Gabor analysis concerning the classical modulation spaces to quasi-Orlicz modulation spaces. We will show that the quasi norm $f \mapsto \|V_{\phi_1} f\|_{L^{\Phi_1,\Phi_2}_{(\omega)}}$ and $f \mapsto \|V_{\phi_2} f\|_{W(L^{\Phi_1,\Phi_2}_{(\omega)})}$ are equivalent when ω is a moderate weight on $\mathbf{R}^{2d}$ and ϕ_1, ϕ_2 are suitable. We will prove that the analysis operator C_{ϕ_1} is continuous from $M^{\Phi_1,\Phi_2}_{(\omega)}(\mathbf{R}^d)$ into $\ell^{\Phi_1,\Phi_2}_{(\omega)}(\mathbf{Z}^{2d})$, and that the corresponding synthesis operator are continuous from $\ell^{\Phi_1,\Phi_2}_{(\omega)}(\mathbf{Z}^{2d})$ to $M^{\Phi_1,\Phi_2}_{(\omega)}(\mathbf{R}^d)$. In the end we will present some consequences of the previous results.

For $p \in (0, \infty]$, let $\Phi_p(t) = \frac{t^p}{p}$ when $p < \infty$, and set $\Phi_\infty(t) = 0$ when $0 \leq t \leq 1$ and $\Phi_\infty(t) = \infty$ when $t > 1$. Then it is well-known that $L^{\Phi_p,\Phi_q}_{(\omega)} = L^{p,q}_{(\omega)}$ with equality in quasi-norms. Hence the family of quasi-Orlicz spaces contain the usual Lebesgue spaces and mixed quasi-normed spaces of Lebesgue types.

This is a joint work with Joachim Toft, Elmira Nabizadeh and Serap Öztop.

References

[1] H. G. Feichtinger *Modulation spaces on locally compact abelian groups.* Technical report, University of Vienna, Vienna, 1983; also in: M. Krishna, R. Radha, S. Thangavelu (Eds) Wavelets and their applications, Allied Publishers Private Limited, NewDehli Mumbai Kolkata Chennai Nagpur Ahmedabad Bangalore Hyderbad Lucknow, 2003, pp. 99–140.

[2] H. G. Feichtinger *Modulation spaces: Looking back and ahead.* Sampl. Theory Signal Image Process. **5** (2006), 109–140.

[3] Y. V. Galperin, S. Samarah, *Time-frequency analysis on modulation spaces $M_m^{p,q}$, $0 < p, q \leq \infty$.* Appl. Comput. Harmon. Anal. **16** (2004), 1–18.

[4] K. H. Gröchenig, *Foundations of Time-Frequency Analysis* . Birkhäuser, Boston, 2001..

[5] P. Harjulehto, P. Hästö, *Orlicz Spaces and Generalized Orlicz Spaces.* Lecture notes in mathematics **2236**, Springer, Cham, 2019.

[6] M. M. Rao, Z. D. Ren, *Theory of Orlicz Spaces.* Marcel Dekker, New York, 1991.

[7] H. Rauhut, *Coorbit space theory for quasi-Banach spaces.* Studia Math. **180** (2007), 237–253.

[8] C. Schnackers, H. Führ, *Orlicz Modulation Spaces.* Proceedings of the 10th International Conference on Sampling Theory and Applications.

[9] M. Signahl, J. Toft , *Mapping properties for the Bargmann transform on modulation spaces.* J. Pseudo-Differ. Oper. Appl. **3**, 1–30.

[10] J. Toft, *Gabor analysis for a broad class of quasi-Banach modulation spaces* in: S. Pilipović, J. Toft (eds), *Pseudo-differential operators, generalized functions.* Operator Theory: Advances and Applications **245**, Birkhäuser, 2015, 249–278.

[11] J. Toft, *Continuity and compactness for pseudo-differential operator with symbols in quasi-Banach spaces or Hörmander classes.* Analysis and Applications, **15** (2016), 353–389.

• The nuclearity of Gelfand-Shilov spaces and kernel theorems

Jasson Vindas

Ghent University

jasson.vindas@ugent.be ABSTRACT

The purpose of this talk is to discuss nuclearity and kernel theorems in the context of global spaces of ultradifferentiable functions with rapid decay at infinity. We introduce general classes of Gelfand-Shilov spaces defined via weight matrices and weight function systems, and characterize when they are nuclear in terms of their defining weight systems. Our results might be regarded as counterparts of the classical nuclearity characterization for Köthe sequence spaces. Our general framework allows for a unified treatment of Gelfand-Shilov spaces defined via weight sequences and

Beurling-Björck spaces described by means of weight functions (of Braun-Meise-Taylor type). Futhermore, our approach is stable under topological tensor products, hence covering anisotropic cases, and leading to new Schwartz-type kernel theorems.

The talk is based on collaborative works with Andreas Debrouwere and Lenny Neyt [1,2].

References

[1] A. Debrouwere, L. Neyt, J. Vindas, *Characterization of nuclearity for Beurling-Björck spaces.* Proc. Amer. Math. Soc. 12 (2020), 5171–5180.

[2] A. Debrouwere, L. Neyt, J. Vindas, *The nuclearity of Gelfand-Shilov spaces and kernel theorems.* Collect. Math. 72 (2021), 203–227.

• The spectral decomposition and the Schrödinger evolution for non-self-adjoint degree-2 Hamiltonians

Joe Viola

Laboratoire de Mathématiques Jean Leray, Université de Nantes:

joseph.viola@univ-nantes.fr:

Certain well-known techniques in quantum mechanics fail when one considers non-self-adjoint Hamiltonians (which appear, for instance, in kinetic theory). Elementary models include the Davies operator / complex harmonic oscillator $-(d/dx)^2 + ix^2$ and the harmonic oscillator with complex shift $-(d/dx)^2 + x^2 + ix$.

In particular, the decomposition in eigenfunctions generally diverges and the Schrödinger evolution is no longer mass-preserving. We will discuss how complex extension of wave-packet decompositions gives us sharp estimates controlling these phenomena. In particular, we will discuss a recent result (joint with B. Mityagin and P. Siegl) on the hypoelliptic Laplacian on the circle, drawing from other works (with A. Aleman, M. Hitrik, K. Pravda-Starov, and J. Sjöstrand) and fundamental classical works by J. Sjöstrand and L. Hörmander.

• Scaling Limit of Modulation Spaces and Their Applications

Baoxiang Wang

School of Mathematical Sciences, Peking University, Beijing 100871, PR of China
Email: wbx@math.pku.edu.cn

Modulation spaces $M^s_{p,q}$ were introduced by Feichtinger in 1983. Bényi and Oh in 2020 defined a modified version to Feichtinger's modulation spaces for which the symmetry scalings are emphasized for its possible applications in PDE. By carefully investigating the scaling properties of modulation spaces and their connections with Bényi and Oh's modulation spaces, we introduce the scaling limit versions of modulation spaces, which contains both Feichtinger's and Bényi and Oh's modulation spaces. As their applications, we will give a local well-posedness and a (small data) global well-posedness results for nonlinear Schrödinger equation in some scaling limit of modulation spaces, which generalize the well posedness results on modulation spaces and certain super-critical initial data in H^s or in L^p are involved in these spaces. This is a joint work with M. Sugimoto.

• Propagation of Global Analytic Singularities for Schrödinger Equations with Quadratic Hamiltonians

Francis White

AFFILIATION / INSTITUTION: University of California Los Angeles

EMAIL: fwhite@math.ucla.edu

We study the propagation in time of 1/2-Gelfand-Shilov singularities, i.e. global analytic singularities, of tempered distributional solutions of the initial value problem

$$\begin{cases} \partial_t u + q^w(x,D)u = 0 \\ u|_{t=0} = u_0, \end{cases}$$

on $\mathbb{R}^n$, where u_0 is a tempered distribution on $\mathbb{R}^n$, $q = q(x,\xi)$ is a complex-valued quadratic form on $\mathbb{R}^{2n} = \mathbb{R}^n_x \times \mathbb{R}^n_\xi$ with nonnegative real part $\operatorname{Re} q \geq 0$, and $q^w(x,D)$ is the Weyl quantization of q. We prove that the 1/2-Gelfand-Shilov singularities of the initial data that are contained within a distinguished linear subspace of the phase space $\mathbb{R}^{2n}$, called the *singular space of* q, are transported by the Hamilton flow of $\operatorname{Im} q$, while all other 1/2-Gelfand-Shilov singularities are instantaneously regularized. Our result extends the observation of Hitrik, Pravda-Starov, and Viola '18 that this evolution is instantaneously globally analytically regularizing when the singular space of q is trivial.

References

[1] White, F. *Propagation of Global Analytic Singularities for Schrödinger Equations with Quadratic Hamiltonians*, 2021. *arXiv*: 2102.01474

4.11 Special Session 11 - Spectral Theory and Differential Operators

• RESOLVENT ESTIMATES FOR SCHRÖDINGER OPERATORS WITH COMPLEX POTENTIALS

ANTONIO ARNAL and **PETR SIEGL**

AFFILIATION / INSTITUTION: QUEEN'S UNIVERSITY BELFAST, U.K.

EMAIL: aarnalperez01@qub.ac.uk

ABSTRACT Let $H = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V(x)$ be a Schrödinger operator acting in $L^2(\mathbb{R})$, where V is a complex function satisfying certain regularity and growth conditions. We study the asymptotic behaviour of the norm of the resolvent, $\Psi(\lambda) := \| (H-\lambda)^{-1} \|$, when the complex number λ lies near the boundary of the numerical range of H. We shall derive explicit estimates for Ψ along the real and imaginary axes which are, in a certain sense, optimal. We will sketch the proof of the main results, outline extensions to higher dimensions and illustrate the results with examples. This presentation is based on joint work with Petr Siegl.

• Locally eventually positive evolution equations

Sahiba Arora

INSTITUTION: Technische Universität Dresden, Germany

EMAIL: sahiba.arora@mailbox.tu-dresden.de

Consider the problem

$$\begin{aligned} u_t + \Delta^2 u &= 0 \qquad \text{in } \mathbb{R}^n \times [0,\infty) \\ u(x,0) &= u_0(x) \qquad \text{in } \mathbb{R}^n. \end{aligned}$$

It was shown by Gazzola and Grunau in 2008 that if u_0 is positive, continuous, and has compact support, then for every compact set K, there exists a time $t_0 \geq 0$ such that $u(x,t) > 0$ for all $x \in K$ and $t \geq t_0$. This notion, that if the initial datum is positive, then the solution becomes (and stays) positive in a part of the domain after a sufficiently large time is called *local eventual positivity* and has been shown to be exhibited by a variety of differential operators.

Using ideas from the recent theory of eventually positive operator semigroups, we briefly look at some conditions that are sufficient for local eventual positivity of evolution equations. After that, we will see how these results can be applied to concrete differential equations, for instance, the bi-Laplace operator with Dirichlet boundary conditions. Lastly, we will look at some spectral properties of local eventual positivity.

While we restrict ourselves to function spaces during the talk, the results mentioned hold on general Banach lattices.

References

[1] Sahiba Arora, *Locally eventually positive operator semigroups.* To appear in Journal of Operator Theory, Preprint available online at https://arxiv.org/abs/2101.11386.

[2] Elvise Berchio, *On the sign of solutions to some linear parabolic biharmonic equations.* Advances in Differential Equations (2008).

[3] Daniel Daners and Jochen Glück and James B. Kennedy, *Eventually and asymptotically positive semigroups on Banach lattices.* Journal of Differential Equations (2016).

[4] Lucas C. F. Ferreira and Vanderley A. Ferreira, *On the eventual local positivity for polyharmonic heat equations.* Proceedings of the American Mathematical Society (2019).

[5] Alberto Ferrero and Filippo Gazzola and Hans-Christoph Grunau, *Decay and local eventual positivity for biharmonic parabolic equations.* Discrete and Continuous Dynamical Systems (2008).

[6] Filippo Gazzola and Hans-Christoph Grunau, *Eventual local positivity for a biharmonic heat equation in* $\mathbb{R}^n$. Discrete and Continuous Dynamical Systems. Series S (2008).

• On Critical Dipoles in Dimensions $n \geq 3$

S. Blake Allan and Fritz Gesztesy

AFFILIATION / INSTITUTION: Graduate Student, Baylor University

EMAIL: Blake_Allan1@baylor.edu

We reconsider generalizations of Hardy's inequality corresponding to the case of (point) dipole potentials $V_\gamma(x) = \gamma(u,x)|x|^{-3}$, $x \in \mathbb{R}^n \backslash \{0\}$, $\gamma \in [0,\infty)$, $u \in \mathbb{R}^n$, $|u| = 1$, $n \in \mathbb{N}$, $n \geq 3$. More precisely, for $n \geq 3$, we provide an alternative proof of the existence of a critical dipole coupling constant $\gamma_{c,n} > 0$, such that

for all $\gamma \in [0, \gamma_{c,n}]$, and all $u \in \mathbb{R}^n$, $|u| = 1$,

$$\int_{\mathbb{R}^n} d^n x \, |(\nabla f)(x)|^2 \geq \pm\gamma \int_{\mathbb{R}^n} d^n x \, (u, x)|x|^{-3}|f(x)|^2, \quad f \in D^1(\mathbb{R}^n).$$

with $D^1(\mathbb{R}^n)$ denoting the completion of $C_0^\infty(\mathbb{R}^n)$ with respect to the norm induced by the gradient. Here $\gamma_{c,n}$ is sharp, that is, the largest possible such constant, and we discuss a numerical scheme for its computation. Moreover, we discuss upper and lower bounds for $\gamma_{c,n} > 0$.

This quadratic form inequality will be a consequence of the fact

$$\overline{\big[-\Delta + \gamma(u, x)|x|^{-3} \big]\big|_{C_0^\infty(\mathbb{R}^n \setminus \{0\})}} \geq 0 \text{ if and only if } 0 \leq \gamma \leq \gamma_{c,n}$$

in $L^2(\mathbb{R}^n)$ (with $\overline{T}$ the operator closure of the linear operator T).

We also consider the case of multicenter dipole interactions with dipoles centered on an infinite discrete set.

References

[1] S. B. Allan and F. Gesztesy, *On Critical Dipoles in Dimensions* $n \geq 3$. Submitted, arXiv:2101.09457

• UNIFORM CONVERGENCE FOR OPERATORS IN DOMAINS PERFORATED ALONG MANIFOLD

D. Borisov And A. Mukhametrakhimova

Institute Of Mathematics, Ufa Federal Research Center, Ras

borisovdi@yandex.ru

We consider a boundary value problem for the second order scalar differential operator H_ε with variable coefficient in a multi-dimensional domain Ω_ε perforated by small holes distributed along a given manifold. This manifold, denoted by S, is located inside a given domain Ω, and the perforation is made by a set of small closely spaced holes, the union of which is denoted by θ^ε. The sizes of the holes and the distances between them are governed by a small parameter ε. The perforated domain Ω_ε is obtained from Ω by removing the holes θ^ε. The shapes of the holes in θ^ε are arbitrary as well as their distribution along the manifold. The equation we consider in Ω_ε reads as

$$\left(-\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} A_{ij} \frac{\partial}{\partial x_j} + \sum_{j=1}^{n} A_j \frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_j} \overline{A_j} + A_0 - \lambda \right) u_\varepsilon = f$$

for a given complex parameter λ and a given function $f \in L_2(\Omega_\varepsilon)$; the differential expression (without λ) is assume to be symmetric. On the external boundary of the domain we impose the Dirichlet condition as well as on the boundaries of some of the holes, while on the boundaries of remaining holes are subject to a nonlinear Robin condition.

Apart of the classification of the homogenized problems, our main result provides the estimates for the convergence rates and the main feature is that these estimates are uniform in the right hand side f.

• Applications of spectral theory in blood flow modeling problems

Marina Chugunova

Abstract: Babies born with a single functioning heart ventricle instead of the usual two require a series of surgeries during the first few years of life to redirect their blood flow. The resulting circulation, in which systemic venous blood flows directly into the pulmonary arteries, bypassing the heart, is referred to as the Fontan circulation. We develop two mathematical lumped parameter models for blood pressure distribution in the Fontan and normal blood flow circulation. Numerical simulations of the ODE model with physiologically consistent input parameters and cardiac cycle pressure-volume outputs reveal the existence of a critical value for pulmonary resistance above which the cardiac output dramatically decreases.

Joint work with: M.G. Doyle, J.P. Keener, and R.M. Taranets

• Inverse problems for first order differential systems with periodic 2×2 matrix potentials and quasi-periodic boundary conditions.

SONJA CURRIE, THOMAS ROTH and BRUCE WATSON

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG:

Sonja.Currie@wits.ac.za

A generalisation is given of the inverse problem considered in S. Currie, B.A. Watson, T.T. Roth, First order systems in $\mathbb{C}^2$ on $\mathbb{R}$ with periodic matrix potentials and vanishing instability intervals, *Math. Meth.*

Appl. Sci. **38** (2015), 4435–4447. In particular, the self-adjoint first order system, $JY' + QY = \lambda Y$, with integrable, real, symmetric, π-periodic, 2×2 matrix potential Q is considered, where $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. It is shown that all eigenvalues to the above equation with boundary conditions $Y(\pi) = \pm R(\theta)Y(0)$, where $R(\theta)$ is the rotation matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, $\theta \in [0, \pi]$, are double eigenvalues if and only if $Q = rI$ for some real scalar valued integrable function r.

References

[1] Currie, S., Roth, T.T. and Watson, B.A., *Inverse problems for first order differential systems with periodic 2×2 matrix potentials and quasi-periodic boundary conditions.* Mathematical Methods in Applied Sciences.

• A regularization technique for ill-posed problems associated with strong-strip type operators

Matthew A. Fury

Penn State Abington

maf44@psu.edu

Abstract

In this presentation, we will consider Cauchy problems of the form $\frac{du}{dt} = p(A)u$, $0 < t < T$, $u(0) = \varphi$ in a Banach space X where iA generates a C_0 group of bounded linear operators on X and $p(\lambda)$ is a complex polynomial. Since iA generates a C_0 group, the operator A is then a strong strip-type operator whose spectrum $\sigma(A)$ is contained in a closed horizontal strip (or interval), $\{z \in \mathbb{C} \ : \ |\mathrm{Im} z| \leq \theta\}$ of height $\theta \geq 0$. Depending on the polynomial $p(\lambda)$, our original problem may be ill-posed whose solutions (if they exist) do not depend continuously on the initial data. Therefore, we seek estimates of supposed solutions of the problem by defining an approximate well-posed problem. We apply a variation of a regularization by Showalter, defined by $\frac{dv}{dt} = r_\beta(A)v$, $0 < t < T$, $v(0) = \varphi$, $r_\beta(A) = p(A)(I + \beta A^2)^{-N}$ where N is a positive integer depending on the degree of the polynomial $p(\lambda)$. The operator $r_\beta(A)$ may be defined by a functional calculus for strip-type operators and many of the calculations required for regularization follow naturally. Finally, by setting $A = -i(d/dx)$, we apply the theory to significant PDEs including the Schrödinger equation and the linearized Korteweg-de Vries equation both of which are ill-posed in the spaces $X = L^p(\mathbb{R})$, $1 < p < \infty$, if $p \neq 2$.

References

[1] K. A. Ames and R. J. Hughes, *Structural stability for ill-posed problems in Banach space.* Semigroup Forum 70 (2005) 127–145

[2] S. Clark, *Sums of Operator Logarithms.* Quar. J. Math. 60 (2009) 413–427

[3] R. deLaubenfels, *Polynomials of generators of integrated semigroups.* Proc. Am. Math. Soc. 99 (1987) 105–108

[4] M. A. Fury and R. J. Hughes, *Regularization for a class of ill-posed evolution problems in Banach space.* Semigroup Forum 85 (2012) 191–212

[5] H. Kellerman and M. Hieber, *Integrated Semigroups.* J. Funct. Anal. 84 (1989) 160–180

[6] I. V. Mel'nikova, *General theory of the ill-posed Cauchy problem.* J. Inverse Ill-Posed Probl. 3 (1995) 149–171

[7] L. E. Payne, *On stabilizing ill-posed problems against errors in geometry and modeling.* In: Inverse and Ill-posed Problems, Academic Press, San Diego (1987), 399–416.

[8] R. E. Showalter, *The final value problem for evolution equations.* J. Math. Anal. Appl. 47 (1974) 563–572

• Differential-algebraic equations in Hilbert spaces

Hannes Gernandt and **Timo Reis**

Technische Universität Hamburg

hannes.gernandt@tuhh.de

In this talk, we consider differential-algebraic equations of the form

$$\tfrac{\mathrm{d}}{\mathrm{d}t}Ex(t) = Ax(t) + f(t), \quad Ex(0) = Ex_0,$$

where $E : H \to H$ is bounded and $A : H \supseteq \operatorname{dom} A \to H$ is closed and densely defined in a Hilbert space H. Such type of equations appear naturally when a dynamical system is not only governed by partial-differential equations but also by certain conservation laws.

We develop a solution theory based on pseudo-resolvents and linear relations. The basic idea is to construct an underlying operator associated to a given pseudo-resolvent. For this construction, a certain dissipativity assumption on the pseudo-resolvents is used. After comparing our construction to other recent approaches by Trostorff & Waurick [2], and Reis & Tischendorf [1], we show that various examples from physics satisfy our dissipativity assumption.

This talk is based on a joint work with TIMO REIS (Universität Hamburg).

References

[1] T. Reis and C. Tischendorf, *On differential-algebraic equations in infinite dimensions.* J. Evolution Equations 5, pp. 357–385, 2005.

[2] S. Trostorff and M. Waurick, *Frequency domain methods and decoupling of linear infinite dimensional differential algebraic systems.* J. Differential Equations 266, pp. 526–561, 2019.

- ## Maximum and anti-maximum principles – an operator theoretic approach

Sahiba Arora and <u>Jochen Glück</u>

AFFILIATION / INSTITUTION: University of Passau, Faculty of Computer Science and Mathematics

EMAIL: jochen.glueck@uni-passau.de

Let us consider a differential operator A on a function space, say $L^p(\Omega)$, with an eigenvalue $\lambda_0 \in \mathbb{R}$. We study the classical equation

$$(\lambda - A)u = f$$

and are interested in the question whether $f \geq 0$ implies $u \geq 0$ for all λ in a left neighbourhood of λ_0; this is an abstract formulation of a *maximum principle* satisfied by A. Similarly, A is said to satisfy an *anti-maximum principle* if, for λ in a left a neighbourhood of λ_0, the inequality $f \geq 0$ implies $u \leq 0$.

Maximum and anti-maximum principles for concrete differential operators have been studied for a long time and are particularly subtle for differential operators of order higher than 2. In this talk, we present a very general operator theoretic approach in order to characterize the validity of (anti-)maximum principles. This generalizes and unifies various results from the literature, including an earlier operator theoretic approach by Takáč. Moreover, it allows us to easily derive new (anti-)maximum principles for various classes of differential operators for which such results have not been shown before.

References

[1] Peter Takáč, *An abstract form of maximum and anti-maximum principles of Hopf's type.* Journal of Mathematical Analysis and Applications, 201(2):339–364, 1996.

[2] Sahiba Arora and Jochen Glück, *Uniform (anti-)maximum principles and eventual positivity.* Preprint, arXiv:2104.12205v1, 29 pages, 2021.

• STRONG MIXING WITH RATE OF CONVERGENCE FOR OPERATOR SEMI-GROUPS WITH NON-SECTORIAL GENERATOR

Martin Grothaus

Mathematics Department / TU Kaiserslautern

EMAIL: grothaus@mathematik.uni-kl.de ABSTRACT

Motivated by problems from Industrial Mathematics we further developed the concepts of hypocoercivity. The original concepts needed Poincaré inequalities and where made for finite dimensional and linear state spaces. In between we can treat as a state space manifolds or even infinite dimensional state spaces. The condition giving micro- and macroscopic coercivity we could relax from Poincaré to weak Poincaré inequalities. In this talk an overview and many examples are given.

• Isomorphism between one-dimensional and multidimensional finite difference operators

Anton A. Kutsenko

Jacobs University, Bremen, Germany

akucenko@gmail.com

Finite difference operators are widely used for the approximation of continuous ones. It is well known that the analysis of continuous differential operators may strongly depend on their dimensions. We will show that the finite difference operators generate the same algebra, regardless of their dimension.

References

[1] Anton A. Kutsenko, *Isomorphism between one-dimensional and multidimensional finite difference operators.* Communications on Pure & Applied Analysis, 2021, 20 (1) : 359-368. doi: 10.3934/cpaa.2020270

• Stability of spectral characteristics of boundary value problems for 2×2 Dirac type systems

Anton A. Lunyov and Mark M. Malamud

Facebook, Inc., Menlo Park, California

a.a.lunyov@gmail.com

Boundary value problems associated in $L^2([0,1];\mathbb{C}^2)$ with the following 2×2 Dirac type equation

$$L_U(Q)y = -iB^{-1}y' + Q(x)y = \lambda y, \quad B = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \quad b_1 < 0 < b_2, \quad y = \operatorname{col}(y_1, y_2), \tag{6}$$

with a potential matrix $Q \in L^p([0,1];\mathbb{C}^{2\times 2})$, $p \geq 1$, and subject to the regular boundary conditions $Uy := \{U_1, U_2\}y = 0$ has been investigated in numerous papers. If $b_2 = -b_1 = 1$ this equation is equivalent to one dimensional Dirac equation.

In this talk we present recent results concerning the stability property under the perturbation $Q \to \widetilde{Q}$ of different spectral characteristics of the corresponding operator $L_U(Q)$ obtained in a preprint [2] jointly with Mark Malamud. Our approach to the spectral stability relies on the existence of the triangular transformation operators for system (6) with $Q \in L^1$, which was established by us in [1]. The starting point of our investigation is the Lipshitz property of the mapping $Q \to K_Q^{\pm}$, where $K_Q^{\pm}$ are the kernels of transformation operators for system (6). Namely, we prove the following uniform estimate:

$$\|K_Q^{\pm} - K_{\widetilde{Q}}^{\pm}\|_{X_{\infty,p}^2} + \|K_Q^{\pm} - K_{\widetilde{Q}}^{\pm}\|_{X_{1,p}^2} \leq C \cdot \|Q - \widetilde{Q}\|_p, \qquad Q, \widetilde{Q} \in \mathbb{U}_{p,r}^{2\times 2}, \quad p \in [1,\infty],$$

on balls $\mathbb{U}_{p,r}^{2\times 2}$ in $L^p([0,1];\mathbb{C}^{2\times 2})$. It is new even for $\widetilde{Q} = 0$. Here $X_{\infty,p}^2$, $X_{1,p}^2$ are the special Banach spaces naturally arising in such problems. We also obtain similar estimates for Fourier transforms of $K_Q^{\pm}$. Both of these estimates are of independent interest and play a crucial role in the proofs of all spectral stability results discussed in this talk. For instance, as an immediate consequence of these estimates we get the Lipshitz property of the mapping $Q \to \Phi_Q(\cdot, \lambda)$, where $\Phi_Q(x, \lambda)$ is the fundamental matrix of the system (6).

Assuming the spectrum $\Lambda_Q = \{\lambda_{Q,n}\}_{n\in\mathbb{Z}}$ of $L_U(Q)$ to be asymptotically simple, denote by $F_Q = \{f_{Q,n}\}_{|n|>N}$ a sequence of corresponding normalized eigenvectors, $L_U(Q)f_{Q,n} = \lambda_{Q,n}f_{Q,n}$. Assuming *boundary conditions to be strictly regular*, we show that the mapping $Q \to \Lambda_Q - \Lambda_0$ sends $L^p([0,1];\mathbb{C}^{2\times 2})$ either into $\ell^{p'}$ or into the weighted ℓ^p-space $\ell^p(\{(1+|n|)^{p-2}\})$; we also establish its Lipshitz property on compact sets in $L^p([0,1];\mathbb{C}^{2\times 2})$, $p \in [1,2]$. The proof of the second estimate involves as an important ingredient inequality that generalizes classical Hardy-Littlewood inequality for Fourier coefficients. We also show that the mapping $Q \to F_Q - F_0$ sends $L^p([0,1];\mathbb{C}^{2\times 2})$ into the space $\ell^{p'}(\mathbb{Z};C([0,1];\mathbb{C}^2)$ of sequences of continuous vector-functions, and has the Lipshitz property on compacts sets in $L^p([0,1];\mathbb{C}^{2\times 2})$, $p \in [1,2]$.

Note also that the proof of the Lipshitz property of the mapping $Q \to F_Q - F_0$ involves the deep Carleson-Hunt theorem for maximal Fourier transform, while the proof of this property for the mapping $Q \to \Lambda_Q - \Lambda_0$ relies on the estimates of the classical Fourier transform and is elementary in character.

References

[1] A.A. Lunyov and M.M. Malamud, *On the Riesz basis property of root vectors system for* 2×2 *Dirac type operators*. J. Math. Anal. Appl. **441** (2016), pp. 57–103 (arXiv:1504.04954).

[2] A.A. Lunyov and M.M. Malamud, *Stability of spectral characteristics and Bari basis property of boundary value problems for* 2×2 *Dirac type systems*. arXiv:2012.11170.

• Trace formulas for pairs of nonselfadjoint operators with trace class resolvent differences

Mark MALAMUD

malamud3m@gmail.com

ABSTRACT

The talk will be devoted to perturbation determinants and trace formulas for a pair of operators with the trace class resolvent difference. The topic is treated in the framework of boundary triplet approach to the extension theory of symmetric operators. More precisely, the perturbation determinants are expressed by means of the Weyl function and boundary operators. Applications to boundary value problems for ordinary differential operators and elliptic operators in bounded or exterior domains will also be discussed.

Following [2]-[4] we will discuss the existence of complex valued spectral shift function for a pair of completely non-selfadjoint maximal dissipative operators with trace class resolvent difference. For such pairs of operators the Krein type trace formulas are established for a class of operator Lipschitz functions. The proof is substantially relied on the method of double operator integrals.

The problem of existence a real valued spectral shift function such pairs of operators will also be discussed.

The talk is based on our joint works with H. Neidhardt and V. Peller [1]–[4]. Some new recent results in this direction will be discussed too.

References

[1] M.M. Malamud, H. Neidhardt, *Perturbation determinants for singular perturbations.*
Russ. J. Math. Phys. **21**, (2014), 55–98.

[2] M. Malamud, H. Neidhardt, *Trace formulas for additive and non-additive perturbations.*
Adv. Math. **274**, (2015), 736–832.

[3] M. Malamud, H. Neidhardt, V.V. Peller, *Analytic operator Lipschitz functions in the disk and a trace formula for functions of contractions.*
Funct. Anal. Appl. **51** (3), (2016), 185–203.

[4] M.M. Malamud, H. Neidhardt, V.V. Peller, *Absolute continuity of spectral shift.*
J. Funct. Anal. **276**, (2019), 1575–1621.

• On Spectral Mapping Theorems and Asymptotics of Scalar Type Spectral C_0-Semigroups

Marat V. Markin

California State University, Fresno

EMAIL: mmarkin@csufresno.edu

We establish spectral inclusion and mapping theorems for scalar type spectral operators, generalizing their counterparts for normal operators. Thereby, we extend a precise weak spectral mapping theorem along with the *spectral bound equal growth bound condition* and a *generalized Lyapunov stability theorem*, known to hold for C_0-semigroups of normal operators on complex Hilbert spaces, to the more general case of C_0-semigroups of scalar type spectral operators on complex Banach spaces. For such semigroups, we obtain exponential estimates with the best stability constants.

We also extend to a Banach space setting a celebrated characterization of uniform exponential stability for C_0-semigroups on complex Hilbert spaces and thereby acquire a characterization of uniform exponential stability for scalar type spectral and eventually norm-continuous C_0-semigroups. The finer spectrum structure is given itemized consideration.

References [1] Marat V. Markin, *On spectral inclusion and mapping theorems for scalar type spectral operators and semigroups*, arXiv:2002.09087.

[2] Marat V. Markin, *On asymptotics for C_0-semigroups*, arXiv:2107.02832.

• Invariance of the Fredholm Index of Non-Smooth Pseudodifferential Operators

Dr. Christine Pfeuffer

As nearly invertible operators Fredholm operators play an important role in the field of partial differential equations in order to obtain existence and uniqueness results. Hence great effort already was spent to get some conditions for the Fredholmness of pseudodifferential operators. However, there are very few results for the invariance of the Fredholm index of such operators.

In the smooth case Schrohe was able to show under certain conditions, that the Fredholm index of smooth pseudodifferential operators is invariant considered as a map between certain weighted Bessel potential spaces with symbols in the Hörmander-class $S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$.

In applications also non-smooth pseudodifferential operators occur. The goal of this talk is to show the invariance of the Fredholm index for non-smooth pseudodifferential operators with symbols in the class $C^{\tilde{m},s}S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$. To reach this aim we use the main idea of the result from Rabier about the Fredholm index for non-smooth differential operators. The main difficulty is to prove a regularity result for non-smooth pseudodifferential operators needed in the proof.

The talk is based on a joint work with H. Abels.

• Diverging eigenvalues in domain truncations of Schrödinger operators with complex potentials

Iveta Semorádová and Petr Siegl

Czech Technical University in Prague & Czech Academy of Sciences

Iveta.Semoradova@fjfi.cvut.cz

Domain truncations of Schrödinger operators with complex potentials are known to be spectrally exact. However, several examples suggest that additional eigenvalues escaping to infinity seem to be a generic feature. We find conditions on the presence of such eigenvalues and obtain their asymptotic expansions. Our approach also yields asymptotic formulas for diverging eigenvalues in a strong coupling regime for the imaginary part of the potential.

• The spectrum of the Ekman boundary layer problem

Petr Siegl and Borbala Gerhat and Orif Ibrogimov

AFFILIATION / INSTITUTION: Queen's University Belfast

EMAIL: p.siegl@qub.ac.uk

Originating in fluid dynamics, the study of linear stability of an Ekman boundary layer gives rise to a spectral problem for a non-selfadjoint operator matrix family. We present new eigenvalue enclosures for the point spectrum of this family and thereby solve an open problem on the existence of open sets of eigenvalues in domains of Fredholmness posed by L. Greenberg and M. Marletta in 2004.

• Spectra of the Steklov- and Robin-Laplace-problems in bounded, cuspidal domains

Jari Taskinen

It is well-known by works of several authors that the spectrum of the Neumann-Laplace operator may be non-discrete even in bounded domains, if the boundary of the domain has some irregularities. In the same direction, in a paper in 2008 with S.A. Nazarov we considered the Steklov spectral problem in a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$, with a peak and showed that the spectrum may be discrete or continuous depending on the sharpness of the peak. Later, we proved that the spectrum of the Robin Laplacian in non-Lipschitz domains may be quite pathological since, in

addition to countably many eigenvalues, the residual spectrum may cover the whole complex plain.

We have recently complemented this study in two papers, where we consider the spectral Steklov- and Robin-Laplace problems in a bounded domain Ω with a peak and also in a family Ω_ε of domains blunted at the small distance $\varepsilon > 0$ from the peak tip. The blunted domains are Lipschitz and the spectra of the corresponding problems on Ω_ε are discrete. We study the behaviour of the discrete spectra as $\varepsilon \to 0$ and their relations with the spectrum of case with Ω. In particular we find various subfamilies of eigenvalues which behave in different ways (e.g. "blinking" and "stable" families") and we describe a mechanism how the discrete spectra turn into the continuous one in this process.

The work is a co-operation with Sergei A. Nazarov (St. Petersburg) and also Nicolas Popoff (Bordeaux).

• Sturm-Liouville problems with eigenparameter dependent transmission conditions The Pontryagin space case

<u>Bruce Watson</u>, Casey Bartels, Sonja Currie

University of the Witwatersrand

b.alastair.watson@gmail.com

We consider two Sturm-Liouville equations on finite intervals which interact via a transmission matrix. The coefficients of the transmission matrix are generalized Nevanlinna functions of the eigenparameter. It is shown that this problem has various different Pontryagin space formulations (each with their own benefits). Eigenvalue asymptotics are presented as well as bounds on the number of non-real and non-semi-simple eigenvalues. The Hilbert space cases of this work can be found in [1] and [2].

References

[1] C. Bartels, S. Currie, M. Nowaczyk and B.A. Watson, *Sturm-Liouville problems with transfer condition Herglotz dependent on the eigenparameter – Hilbert space formulation.* Integral equations and operator theory, 90 (2018), no. 34, pp.1-20.

[2] C. Bartels, S. Currie and B.A. Watson, *Sturm-Liouville problems with transfer condition Herglotz dependent on the eigenparameter – Eigenvalue asymptotics.* Complex analysis and operator theory, to appear.

• Dispersive Estimates for Schrödinger Equations

Ricardo Weder

Universidad Nacional Autónoma de México

weder@unam.mx ABSTRACT

The importance of the dispersive estimates for Schrödinger equations in spectral theory and in nonlinear analysis will be discussed. Furthermore, the literature on the $L^p - L^{p'}$ estimates will be reviewed, starting with the early results in the 1990 th, and with an emphasis in the results in one dimension. New results will be presented, in $L^p - L^{p'}$ estimates for matrix Schrödinger equations in the half-line, with general self-adjoint boundary condition, and in matrix Schrödinger equations in the full-line with point interactions. In both cases we consider integrable matrix potentials that have a finite first moment.

References

[1] T. Aktosun and R. Weder. *Direct and Inverse Scattering for the Matrix Schrödinger Equation.* Applied Mathematical Sciences **203** , Springer Verlag New York, 2021.

[2] I. Naumkin, R. Weder. *$L^p - L^{p'}$ estimates for matrix Schrödinger equations.* Journal of Evolution Equations **21**, 891-919 (2021). ArXiv 1906.07846 [math-ph].

[3] R. Weder. The L^p boundedness of the wave operators for matrix Schrödinger equations. arXiv 1912.12793 [math-ph].

• Rough coefficients in ordinary differential equations

Rudi Weikard

University of Alabama at Birmingham

EMAIL: weikard@uab.edu

ABSTRACT: We investigate the spectral theory for the system $Ju' + qu = w(\lambda u + f)$ of ordinary differential equations where J is constant invertible skew-hermitian matrix while q is a hermitian and w a non-negative matrix whose entries are distributions of order 0. A major obstacle is the fact that, in general, the unique continuation of solutions of the differential equation is not possible.

This is joint work with K. Campbell, A. Ghatasheh, M. Nguyen, and S. Redolfi

4.12 Special Session 12 - Theory of Superoscillations

• Superoscillations of solutions of arbitrary Schrödinger's equations

Yakir Aharanov

Chapman University

aharanov@chapman.edu

We show that a complete set of eigenvectors of a large class of hermitian operators exhibit the phenomenon of superoscillations. For example, the solutions of any hamiltonian $\frac{p^2}{2m} + V(x)$, for any $V(x)$, indeed have the superoscillation property. We will consider some special examples to demonstrate this result, and in particular, we consider the superoscillations of solutions for any scattering problem.

• Superoscillations: six recent developments

Michael Berry

University of Bristol, Europe

- Noise suppresses superoscillation
- Defining superoscillation for real functions
- Differentiations suppress superoscillations
- Aharonov-Bohm streamlines contain unexpected superoscillations
- Boundaries of superoscillatory regions are zeros of the quantum potential
- Superoscillations are much rarer for vector waves

• Optical superoscillations in time domain

Denys Bondar

Tulane University

dbondar@tulane.edu

Superoscillations (SO) refer to the phenomenon where a band-limited signal can locally oscillate faster than its fastest Fourier component. However, practical explorations of SOs have so far focused on spatial oscillations to improve resolution in microscopy. In the time domain, on the contrary, the experimental development has been very limited: SO were synthesized for radio frequencies, acoustic, and envelopes of near-infrared laser pulses only. The time-domain SO of the electric field of light (not just the pulse envelope) have not been achieved yet. In this talk, I will review our on-going theoretical and experimental effort, generously supported by the Keck Foundation, to enable new optical technologies of superoscillating spectroscopy and superoscillating polarimetry.

• Supergrowth as the dual of superoscillations: Implications for enhanced optical imaging

Andrew N. Jordan

Institute for Quantum Studies, Chapman University, Orange, CA 92866, USA

EMAIL: prof.andrew.jordan@gmail.com

Superoscillations is the phenomenon of the local wavenumber exceeding the highest wavenumber in a bounded Fourier series. I will introduce the concept of supergrowth - when the local rate of growth of a function exceeds the highest wavenumber in a bounded Fourier series [1]. Superoscillations and supergrowth may be viewed as the real and imaginary part of the weak value of momentum (in a quantum mechanical context), where the pre-selection is on a super-state, and the post-selection is on a position. I will demonstrate the supergrowth has great promise for enhanced imaging via point spread function engineering. The enhanced sensitivity also comes with must greater intensity than superoscillations, permitting enhanced signal-to-noise ratio in the image reconstruction. Sketches of ongoing experiments demonstrating this concept will be given.

References

[1] Andrew N. Jordan, *Superresolution using supergrowth and intensity contrast imaging.* Quantum Stud.: Math. Found. **7**, 285–292 (2020).

• On conservation laws in quantum mechanics

Sandu Popescu

S.Popescu@bristol.ac.uk

Conservation laws are one of the most important aspects of nature. As such, they have been intensively studied and extensively applied, and are considered to be perfectly well established. We, however, raise fundamental question about the very meaning of conservation laws in quantum mechanics. We argue that, although the standard way in which conservation laws are defined in quantum mechanics is perfectly valid as far as it goes, it misses essential features of nature and has to be revisited and extended. Superoscillations are at the core of the effects that lead to this conclusion.

• Superoscillations, supershifts and infinite order differential operators

D. Alpay, F. Colombo, S. Pinton, Irene Sabadini and D. C. Struppa

Politecnico di Milano

irene.sabadini@polimi.it

In the recent years, superoscillating functions, that appear for example in weak values in quantum mechanics, have become an interesting and independent field of research in complex analysis and in the theory of infinite order differential operators. We shall discuss some infinite order differential operators acting on entire functions which naturally arise in the study of superoscillating functions, see e.g. [1]. Such operators can be associated with a new binary operation on the frequencies that we call relativistic sum (inspired by the relativistic sum of the velocities). To show that some sequences of functions preserve the superoscillatory behavior, it is of crucial importance to prove that their associated infinite order differential operators act continuously on some spaces of entire functions with growth conditions. We will also discuss how infinite order differential operators can be extended to the setting of entire hyperholomorphic functions.

References

[1] Y. Aharonov, F. Colombo, I. Sabadini, D. C. Struppa, J. Tollaksen, *The mathematics of superoscillations.* Mem. Amer. Math. Soc. 247 (2017), no. 1174.

[2] D. Alpay, F. Colombo, S. Pinton, I. Sabadini, *Holomorphic functions, relativistic sum, Blaschke products and superoscillations.* Preprint 2020.

[3] D. Alpay, F. Colombo, S. Pinton, I. Sabadini, D. C. Struppa, *Infinite order differential operators acting on entire hyperholomorphic functions.* The Journal of Geometric Analysis. https://doi.org/10.1007/s12220-021-00627-y.

• Schrödinger evolution of superoscillations and supershifts.

A unified approach

Peter Schlosser
Institute of Applied Mathematics
Graz University of Technology

Abstract

One main question in the research field of superoscillations is the persistence in the time evolution with respect to the Schrödinger equation. In particular, choosing a superoscillatory function as initial condition, we ask the question whether the solution of the time dependent Schrödinger equation is still superoscillating.

Until now, this problem is only solved for specific potentials, where in particular the corresponding Green's function is known explicitely. The novelty of our unified approach is that we only assume regularity (holomorphicity) and (exponential) growth conditions on the Green's function, but do not need its explicit form.

However, in this general context the notion of supersocillations is too narrow to prove time persistence. Hence we are forced to extend to the more general concept of supershift. Moreover, we point out a similarity of this supershift property and the holomorphicity of the involved functions in the frequence variable.

References

[1] Y. Aharonov, J. Behrndt, F. Colombo, P. Schlosser, *A unified approach to Schrödinger evolution of superoscillations and supershifts*, `arXiv:2102.11795`

• A new method to generate superoscillating functions and supershifts

T. Shushi, Y. Aharonov, F. Colombo, I. Sabadini, D.C. Struppa, J. Tollaksen

AFFILIATION / INSTITUTION: Ben-Gurion University of the Negev

EMAIL: tomershu@bgu.ac.il

ABSTRACT

Superoscillations are band-limited functions that can oscillate faster than their fastest Fourier component. These functions (or sequences) appear in weak values in quantum mechanics and in many fields of science and technology such as optics, signal processing and antenna theory. We introduce a new method to generate superoscillatory functions that allows us to construct explicitly a very large class of superoscillatory functions.

References

[1] Aharonov, Y., Colombo, F., Sabadini, I., Shushi, T., Struppa, D. C., & Tollaksen, J. (2021). *A new method to generate superoscillating functions and supershifts.* Proceedings of the Royal Society A. Accepted.

• Superoscillations and supershifts

facing optics or time-dependent Schrödinger evolution

Alain Yger

University of Bordeaux

yger@math.u-bordeaux.fr

In this talk, I will present under various facets the concept of superoscillating sequence and, in view of the evolution problem to which such concept is confronted with, that of supershift. I will insist on the necessity to enlarge such concept to the hyperfunction setting, in order that singularities which occur through evolution (for example, when such evolution is ruled by the time - dependent Schrödinger equation, where the potential is such that ones remains within the range of Fresnel type operators, as it is the case for the harmonic oscillator) does not contradict the supershift approach. On the other hand, periodic potentials (compared to polynomial potentials as quadratic ones which put us back to Fresnel type evolution operators) look as natural candidates for time - dependent Schrödinger evolution that would escape the supershift approach. Such considerations were initially motivated by optics, more precisely by the optical computation of Gauss sums through the well konwn Talbot carpet. Despite the fact that the so-called superoscillation phenomenom occurs with compact support in time (that is not in contradiction with the uncertainty principle), I will insist in this talk with somehow strange consequences of the superoscillation concept : optical computation of Gauss sums illustrate that, as well as the multifractal description of singularities such as those of the Riemman's function.

References

[1] F. Colombo, I. Sabadini, D. Struppa, A. Yger, Gauss sums, superoscillations and the Talbot carpet, J. Math. Pures et Appliquées, **147**, 2021

[2] F. Colombo, I. Sabadini, D. Struppa, A. Yger, Superoscillating sequences and hyperfunctions, Publications RIMS, Volume 55, Issue 4, 2019.

[3] F. Colombo, I. Sabadini, D. Struppa, A. Yger, Superoscillating sequences and supershifts for families of generalized functions, arXiv:1912.01057, submitted

• The Technology of Optical Superoscillations

Nikolay I. Zheludev

University of Southampton, UK & Nanyang Technological University, Singapore

niz@orc.soton.ac.uk

We report on recent advances in applications of deep learning and superoscillatory light to far-field non-destructive imaging and metrology with deeply subwavelength resolution.

4.13 Special Session 13 - General Session for Contributed Papers

• An introduction to interpolation Spaces

Rizwan Anjum

The theory if interpolation spaces is relatively new area in functional analysis with applications in analysis. In this talk, I will present an introduction to interpolation theory with some examples. I will also discuss the close relation between rearrangement-invariant function spaces and interpolation spaces.

• The trifecta of Hilbert spaces on Unit Disc

HIMANSHU SINGH

University of South Florida, USA

himanshu4@usf.com ABSTRACT

The Hilbert spaces are common. But the direct connection between them is rare. The aim of this paper is to establish a direct relation among the three Hilbert spaces, that are Hardy, Bergman and Dirichlet, without defining any of the Hilbert space in *weighted* sense. In order to accomplish this goal, this paper develops the Littlewood-Paley type Identities for Bergman and Dirichlet space. After defining these identities, the vision of connecting all the three Hilbert spaces via a direct connection is achieved.

• Liouville Weighted Composition Operators over the Fock space

HIMANSHU SINGH

University of South Florida, USA

himanshu4@usf.com ABSTRACT

In this presentation introduces Liouville Weighted Composition Operators,

which are formally given as

$$A_{f,\phi} g = \nabla g(\phi) \cdot D\phi \cdot f,$$

over the Fock space, $F^2(\mathbb{C}^n)$, where $f : \mathbb{C}^n \mapsto \mathbb{C}^n$ and $\phi : \mathbb{C}^n \mapsto \mathbb{C}^n$ are entire functions over $\mathbb{C}^n$.

This discussion will examine various function theoretic properties of these operators, including *closability*, *boundedness* and *compactness*, as well as estimates on the *essential norm* of the operators.

This work was performed in collaboration with *Drs. Joel A. Rosenfeld*[1] and *Benjamin P. Russo*[2], and was funded by *AFOSR Award FA9550-20-1-0127* and *NSF award ECCS-2027976.*

[1]PhD Advisor, University of South Florida, Department of Mathematics and Statistics, `rosenfeldj@usf.edu`

[2] Farmingdale State College (SUNY), Department of Mathematics, `russobp@farmingdale.edu`