Mathematics, Signal Processing and Linear Systems

New Problems and Directions

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Abstracts

Linear Stochastic Systems and a new family of Topological Algebras

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Motivated by the theory of non commutative stochastic distributions, we introduce algebras which are inductive limits of Banach spaces and carry inequalities which are counterparts of the inequality for the norm in a Banach algebra. We then define an associated Wiener algebra, and prove the corresponding version of the well-known Wiener theorem. Finally, we consider factorization theory in these algebra, and in particular, in the associated Wiener algebra. The talk is based on joint works with Palle Jorgensen and Guy Salomon.

References

D. Alpay and G. Salomon. *On algebras which are inductive limits of Banach spaces.* Integral Equations and Operator Theory, vol. 83, Issue 2 (2015), pp. 211-229.
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The Bounded Real Lemma for nonrational operator-valued transfer functions

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The Bounded Real Lemma characterizes in terms of the system matrix $\left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right]$ when a rational matrix function $F(\lambda)$ realized in the form $F(\lambda) = D + \lambda C(I - \lambda A)^{-1}B$ has analytic continuation to a Schur-class function (i.e., holomorphic on the unit disk $\mathbb{D}$ with contractive matrix values). The characterization is in terms of existence of a positive definite solution H of a Linear Matrix Inequality (the Kalman-Yakubovich-Popov (KYP) inequality). Recent extensions ([1], [2]) of this result to nonrational operator-valued functions involve a number of different settings (with or without controllability/observability or stability assumptions, with or without strict inequalities). In these various settings sometimes unbounded solutions of the KYP inequality are required while in other instances bounded solutions suffice. In this talk we show how an operator-theoretic approach reconciles and unifies these diverse results.

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Hardy Hodge decomposition of vector fields

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We show that on a smooth compact hypersurface embedded in R^n, a L_p vector with $1 < p < \infty$ field is uniquely the sum of the trace of a harmonic gradient in the unbounded component of the complement, of the trace of a harmonic gradient in the bounded component, and of a tangent divergence free vector field. On a Lipschitz hypersurface, the result continues to hold but for restricted range of p. The decomposition generalizes both the Helmholtz-Hodge decomposition on amanifold and the decomposition of a complex function on a curve in the plane as the sum of two Hardy functions. This is joint work with D. Pei and Q. Tao.

Arrhythmia Classification Feature Extraction with Symlet Wavelets

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The electrocardiogram (ECG) is an efficient tool to assess heart health and diagnose heart arrhythmias (irregular heartbeats) by capturing the heart′s electrical activity. Atrial Fibrillation (AF) is one the most common serious arrhythmias and is therefore of particular interest to diagnose. We focus on training a classifier to automatically detect if an arrhythmia is present in an ECG and distinguish whether or not the arrhythmia is AF. To accomplish this, we first preform 6-level wavelet decomposition with a Symlet5 wavelet on each ECG signal to help isolate and detect the QRS complex, T-wave, and P-wave components of the ECG (these components play a key role in diagnosing arrhythmias) [1]. We then extract different features associated with each of these components as well as use several summary statistics of the wavelet coefficients themselves as features for our model [2]. Finally, we reduce the feature space with principal component analysis (PCA) to the most significant principal components from which we will train a multinomial logistic regression classifier. We train our model on a labelled data set (collected from the portable AliveCor ECG device) containing the following 4 classes: normal, AF, other arrhythmia, or noisy. The data set contains 8528 single lead ECG recordings lasting from 9 to 60 seconds, each of which were sampled at 300 Hz. Wavelet decomposition was performed in Python with the Pywavelet package and the statistical analysis and model building was done in R.

References

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Couplings of differential operators

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In this talk we discuss orthogonal sums of ordinary and partial differential operators on unions of bounded and unbounded intervals and domains, respectively. It will be shown in which way the natural selfadjoint realization on the union of the intervals or domains is related to the orthogonal sums of the individual operators, and the resolvent differences are expressed via a Krein type formula in terms of Titchmarsh-Weyl functions or Dirichlet-to-Neumann maps.

On the outlying eigenvalues of a polynomial in large independent random matrices

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Consider a selfadjoint polynomial $P(X,Y)$ in two noncommuting selfadjoint indeterminates. In this talk, we will discuss the asymptotic eigenvalue behavior of the random matrix $P(A_N, B_N)$, where A_N and B_N are independent random matrices and the distribution of B_N is invariant under conjugation by unitary operators. We assume that the empirical eigenvalue distributions of A_N and B_N converge almost surely to deterministic probability measures μ and ν, respectively. In addition, the eigenvalues of A_N and B_N are assumed to converge uniformly almost surely to the support of μ and ν, respectively, except for a fixed finite number of fixed eigenvalues (spikes) of A_N. It is known that the empirical distribution of the eigenvalues of $P(A_N, B_N)$ converges to a certain deterministic probability measure Π_P, which is described in terms of Voiculescu's free probability. In addition, when there are no spikes, the eigenvalues of $P(A_N, B_N)$ converge uniformly almost surely to the support of Π_P. When spikes are present, we show that the eigenvalues of $P(A_N, B_N)$ still converge uniformly to the support of Π_P, with the possible exception of certain isolated outliers whose location can be determined in terms of μ, ν, P and the spikes of A_N. A similar result is known in the case when B_N is a Wigner matrix. The relation between outliers and spikes is described using the operator-valued subordination functions of free probability theory. These results extends known facts from the special case in which $P(X,Y) = X + Y$ (see [2, 3]). The talk is based on the preprint [1].

References

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Quasi-monogenic functions

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The Hilbert transform and the Riesz transforms are important operators in Harmonic and Clifford analysis. The Riesz-Hilbert transform is the phase of the Dirac operator. We construct a class of first order operators with Fourier symbol

$$\mathcal{F}D_{\mathcal{H}} = |\underline{\omega}|h(\underline{\omega}),$$

where

- h is an L^2-multiplier.
- The associated transform should be invertible.
  We ensure this by $h(\underline{\omega})\overline{h(\underline{\omega})}^{\mathbb{CH}} = 1$.
- The Riesz-Hilbert transform (and it's Fourier symbol) should be (para-) vector-valued.
- The Riesz-Hilbert transform will be self-adjoint if $\overline{h} = -h$.

A function u is *left quasi-monogenic* in $\mathbb{R}^{n+1}$ iff $(\partial_{x_0} + D_{\mathcal{H}})u = 0$.

We consider a construction of quasi-monogenic functions and an associated Hardy space decomposition and some examples of generalized Riesz-Hilbert transforms associated to quasi-monogenic functions and generalized Dirac operators.

Linear systems associated with regular noncommutative formal power series

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Let $\mathbb{F}_d^+$ be the unital free semigroup generated by d letters $\{1,\ldots,d\}$, and let $p(z) = \sum_{\alpha\in\mathbb{F}_d^+} p_\alpha z^\alpha$ be a formal power series in d formal noncommuting variables $z = (z_1,\ldots,z_d)$ with scalar coefficients $p_\alpha \in \mathbb{C}$ such that $p_\emptyset = 0$, $p_\alpha > 0$ if $|\alpha| = 1$, and $p_\alpha \geq 0$ for all $\alpha \in \mathbb{F}_d^+$. By z^α (and also $\mathbf{A}^\alpha$ for a d-tuple of Hilbert space operators) we mean

$$z^\alpha = z_{i_N} z_{i_{N-1}} \cdots z_{i_1} \quad \text{and} \quad \mathbf{A}^\alpha = A_{i_N} A_{i_{N-1}} \cdots A_{i_1} \quad \text{if} \quad \alpha = i_N i_{N-1} \cdots i_1 \in \mathbb{F}^d.$$

For a fixed integer $n \geq 1$, we associate with p the positive weight $\boldsymbol{\omega} = \{\omega_{p,n;\alpha}\}_{\alpha\in\mathbb{F}_d^+}$ via the equality

$$\sum_{\alpha\in\mathbb{F}_d^+} \omega_{p,n;\alpha}^{-1} z^\alpha = \sum_{j=0}^{\infty} \begin{pmatrix} n+j-1 \\ j \end{pmatrix} (p(z))^j,$$

and then the weighted linear system

$$\begin{cases} x(1\alpha) &= \dfrac{\omega_{p,n;\alpha}}{\omega_{p,n;1\alpha}} A_1 x(\alpha) + \dfrac{1}{\omega_{p,n;1\alpha}} B_{1,\alpha} u(\alpha) \\ \vdots & \qquad \vdots \qquad\qquad\qquad \vdots \\ x(d\alpha) &= \dfrac{\omega_{p,n;\alpha}}{\omega_{p,n;d\alpha}} A_d x(\alpha) + \dfrac{1}{\omega_{p,n;d\alpha}} B_{d,\alpha} u(\alpha) \\ y(\alpha) &= Cx(\alpha) + \omega_{p,n;\alpha}^{-1} D_\alpha u(\alpha). \end{cases} \tag{1}$$

with a d-tuple $\mathbf{A} = (A_1, \dots, A_d)$ of state space operators $A_j : \mathcal{X} \to \mathcal{X}$, the state-output operator $C : \mathcal{X} \to \mathcal{Y}$, and a family of colligation matrices and the family of input spaces indexed by $\alpha \in \mathbb{F}_d^+$:

$$\mathbf{U}_\alpha = \begin{bmatrix} A & \widehat{B}_\alpha \\ C & D_\alpha \end{bmatrix} : \begin{bmatrix} \mathcal{X} \\ \mathcal{U}_\alpha \end{bmatrix} \to \begin{bmatrix} \mathcal{X}^d \\ \mathcal{Y} \end{bmatrix}, \quad \text{where} \quad A = \begin{bmatrix} A_1 \\ \vdots \\ A_d \end{bmatrix}, \ \widehat{B}_\alpha = \begin{bmatrix} B_{1,\alpha} \\ \vdots \\ B_{d,\alpha} \end{bmatrix}. \tag{2}$$

Associated with the system (1) are the observability operator $\mathcal{O}_{p,n;C,\mathbf{A}} \colon x \mapsto \sum_{\alpha \in \mathbb{F}_d^+} (\omega_{p,n;\alpha}^{-1} C \mathbf{A}^\alpha x) z^\alpha$ and the family of transfer functions. Under certain metric constraints imposed on the colligation matrices (2). the operator $\mathcal{O}_{p,n;C,\mathbf{A}}$ maps the state space $\mathcal{X}$ into (or onto) the weighted Hardy-Fock space $H^2_{\boldsymbol{\omega}_{p,n}}(\mathbb{F}_d^+) = \left\{ \sum_{\alpha \in \mathbb{F}_d^+} f_\alpha z^\alpha \colon \sum_{\alpha \in \mathbb{F}_d^+} \omega_{p,n;\alpha} \cdot |f_\alpha|^2 < \infty \right\}$, while transfer functions are Bergman-inner power series in $H^2_{\boldsymbol{\omega}_{p,n}}(\mathbb{F}_d^+)$ and provide a version of the Beurling-Lax theorem in this space.

Non-Commutative Harmonic Analysis for ternary Clifford algebras

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The Dirac operator in standard Clifford Analysis describes Fermions as well as general SU(2)-symmetries. But, motivated by N-body systems, there is a high interest in the study of SU(n)-symmetries, for instance, the SU(3) symmetries arise from the 3-body problem or from diffraction in optics. In this talk we look to an approach to study Dirac operators co-variant under such symmetries based on ternary Clifford algebras. We describe fractional derivatives and higher order decompositions of the Laplacian with respect to these algebras and provide the basic tools for a function theory in this setting.

Semicircular Elements Induced by Orthogonal Projections Signals with Statistical Data?

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Starting from mutually orthogonal integer-many projections in a fixed C^*-probability space, we construct a corresponding Banach $*$-probability space generated by semicircular elements induced by the projections. The construction, itself, is the main result of this talk, and it may provide a certain way to establish signal processing, equipped with, or determined by the semicircular law.

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An introduction to Aharonov-Berry superoscillations

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Aharonov-Berry superoscillations are band-limited functions that can oscillate faster than their fastest Fourier component. This is mathematically possible because the coefficients of the linear combinations of the band limited components depend on the number of components. This phenomenon was discovered in the context of quantum physics, but it has important applications in several research fields, including metrology, antenna theory, and superresolution in optics. In this talk we give an introduction to the mathematical theory developed in the recent years.

On Szegő's theorem for polynomials orthogonal in an indefinite metric

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We will discuss a non-classical case of orthogonal polynomials on the unit circle (abbreviated by OPUC). Namely, it is the case when only a finite number of Verblunsky coefficients lie outside the closed unit disk. In particular, we will consider how this case is related to pseudo-Carathéodory functions, which are an efficient tool to approach various problems arising in digital signal processing and circuits and systems theory.

The main goal will be to show how to carry over the OPUC machinery to this nonstandard situation. As a consequence, we will be able to prove Szegő's theorem in the considered case and get asymptotic results for the corresponding orthogonal polynomials.

References

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Spectral factorization of invertible semi-separable systems via inner-outer factorization

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The talk elucidates the connection between inner-outer factorization and (general) spectral factorization for a (mixed causal-anticausal) semi-separable (or quasi-separable) system. A general spectral factorization of an invertible system splits the system in a product of an outer system with a conjugate outer system. It turns out that such a spectral factorization can be obtained from a single inner-outer factorization based on a well-chosen realization of the original mixed system. Necessary and sufficient conditions for existence also follow. The result leads to an algorithm for spectral factorization that uses only orthogonal transformations.

Reference

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Matrix convex sets without absolute extreme points

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Let $M_n(\mathbb{S})^g$ denote g-tuples of $n \times n$ complex self-adjoint matrices. Given tuples $X = (X_1, \dots, X_g) \in M_{n_1}(\mathbb{S}^g)$ and $Y = (Y_1, \dots, Y_g) \in M_{n_2}(\mathbb{S})^g$, a matrix convex combination of X and Y is a sum of the form

$$V_1^* X V_1 + V_2^* Y V_2 \qquad V_1^* V_1 + V_2^* V_2 = I_n$$

where $V_1 : M_n(\mathbb{R}) \to M_{n_1}$ and $V_2 : M_n(\mathbb{R}) \to M_{n_2}$ are contractions. Matrix convex sets are sets which are closed under matrix convex combinations. A key feature of matrix convex combinations is that the g-tuples X, Y, and $V_1^* X V_1 + V_2^* Y V_2$ do not need to have the same size. As a result, matrix convex sets are a dimension free generalization of convex sets.

While in the classical setting there is only one good notion of an extreme point, there are three natural notions of extreme points for matrix convex sets: Euclidean, matrix, and absolute extreme points. A central goal in the theory of matrix convex sets is to determine if one of these notions of extreme points for matrix convex sets is minimal with respect to spanning.

Matrix extreme points are the most restricted type of extreme point known to span matrix convex sets; however, they are not necessarily the smallest set which does so. Absolute extreme points, a more restricted type of extreme points that are closely related to Arveson's boundary, enjoy a strong notion of minimality should they span. However, until recently it has been unknown if general matrix convex sets are spanned by their absolute extreme points.

This talk will give a class of closed bounded matrix convex sets which do not have absolute extreme points. The sets considered are non-commutative sets, K_X, formed by taking matrix convex combinations of a single tuple X. In the case that X is a tuple of compact operators with no nontrivial finite dimensional reducing subspaces, K_X is a closed bounded matrix convex set with no absolute extreme points. This result shows that matrix convex sets may fail to be spanned by their absolute extreme points and suggests that there is no natural notion of extreme point for matrix convex sets which is minimal with respect to spanning.

THOUGHTS ON OPTIMAL CONTROL

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The purpose of this talk is to outline a, seemingly new, approach to a wide variety of optimal control problems for linear, causal, time-invariant systems. This approach has the advantages of not being restricted to finite-dimensional systems, and has extensions to optimization problems for various classes of transfer functions, including positive real and bounded real functions.

Control theory has developed over the years into a very broad subject, making it difficult to get a good grasp on the various aspects of the subject and the way they are related. Even restricting ourselves to linear, time-invariant, systems, this difficulty is enhanced by the wide choice of system descriptions. We can choose to use external, that is input/output descriptions, or internal descriptions, namely models that explain the external behavior of the given system. Models are far from unique and state space models are but one of many. In fact, even if state space is the form we may prefer for computational purposes, it may not be the best representation for the analysis, and solution, of most control problems. There is another choice to be made due to the possibility of passing on, using various transforms, from the time domain to the frequency domain. In many cases, the frequency domain provides a setting with a richer functional structure that facilitates the solution of the problems of interest.

Apart from the setting, there is a wide variety of control problems to be considered. These include robust stabilization, model reduction, optimal regulator and estimation problems. In order to gain a good understanding of the subject, it is not enough to find a solution to any particular problem. It is of utmost importance to also clarify the relations between different aspects of the theory. Thus, whenever possible, we indicate different approaches to a particular result.

The choice we made is to work mostly in the frequency domain setting, in particular in the use of vectorial Hardy spaces $H^2_\pm$ as signal spaces and co-invariant subspaces as state spaces for the system. This choice has the additional advantage of being able to use the algebraic theory, emphasizing the polynomial module structure, as a guide.

Although the technicalities of polynomial model based system theory for discrete time linear systems over an arbitrary field are vastly different from the Hardy space based theory for some classes of continuous-time systems there are strong algebraic similarities. These, with the help of heavy analytic tools, can be used to extend the algebraic approach to a wide variety of optimal control and estimation problems for several classes of, not necessarily rational, analytic functions. Due to the underlying Hilbert space structure of Hardy spaces, the treatment of optimal control problems are greatly simplified. As we shall try to show, this has the potential of leading to a grand unification of optimal control theory. Thus, (doubly) coprime factorizations over $H^\infty_\pm$ play a central role. Although many of the theorems we use are true in appropriate infinite dimensional setting, presently, we shall deal mostly with the finite dimensional case. Other than studying infinite dimensional systems, this opens up the possibility of extending the methods to other settings as, for example to special classes of systems (positive real, bounded real). Another challenging direction for future research is to extend optimal control theory to deal with complexity, that is, to networks of systems, using local optimality results for the nodes as well as the interconnection data.

Here, in telegraphic style, is an outline of the suggested approach to optimal control theory for stable systems. It is based on [7].

- Describe the optimization problems in the time domain from the input/output point of view. Choose the signal spaces to be $L^2_{(-\infty,\infty)}$ spaces. Introduce the left and right translation groups. Describe the input/output map in terms of a convolution integral with an appropriate kernel. Characterize causality and boundedness.
- Use the Fourier-Plancherel transform, and the Paley-Wiener theorems, to reformulate the setting to that of the Hardy spaces $H^2_\pm$ setting. Introduce in the Hardy spaces the $H^\infty_\pm$-module structure.
- Discuss stability, transfer functions. Identify the restricted input/output map with a Hankel operator. Characterize Hankel operators as H^∞_--module homomorphisms with respect to the H^∞_--module structures of the Hardy spaces $H^2_\pm$.
- Use the Beurling-Lax characterizations of invariant subspaces, See [1, 10]. Relate the kernel and image of the Hankel operator to the Douglas-Shapiro-Shields factorizations, that is coprime factorizations over H^∞_-, see [2, 4].
- Discuss how inner functions are derived through spectral factorizations, or alternatively, by solving a Lyapunov equation or, alternatively, a homogeneous Ricatti equations.
- Use the Kalman approach to realizations as factorizations to identify the restricted Hankel operator, that is the map from the orthogonal complement of the kernel to the range, as a reachability operator. Both these subspaces are called model spaces and play a central role as state spaces.
- Explain the connection between Hankel operators and intertwining maps, that is $H^\infty_\pm$-homomorphisms, between model spaces.
- Explain how an $H^\infty_\pm$-isomorphism can be inverted by solving a Bezout equation, as in [3], or, even better, by embedding an intertwining relation in a doubly coprime factorization.
- Apply this invertibility procedure to the solution of optimal control problems.
- State the solution in terms of a state space realization.

So far we outlined the frequency domain solution to the optimal control problem for the case of a stable, H^∞_+ transfer function.. This can be taken as a intermediate step towards the analysis of the general, not necessarily stable, case. We present the basic ideas in the same condensed style as before. Some of the ideas and results presented owe much to a cooperation with Raimund Ober in the early 1990s and a long term one with Uwe Helmke, culminating in [8].

- Given the strictly proper transfer function $G(s)$, construct a normalized coprime factorizations of the form
$$G = N_r M_r^{-1} = M_\ell^{-1} N_\ell, \tag{3}$$
with all factors in H^∞_+ and the normalization conditions
$$\begin{pmatrix} M_r^* & N_r^* \\ -N_\ell & M_\ell \end{pmatrix}\begin{pmatrix} M_r & -N_\ell^* \\ N_r & M_\ell^* \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$
satisfied.

- Derive state space representatins for all the the factors, see [9].
- Derive stabilizing controllers, having the coprime factorization representations $K = U_r V_r^{-1} = V_\ell^{-1} U_\ell$ by solving the Bezout equations $V_\ell M_r + U_\ell N_r = I$ and $M_\ell V_r + N_\ell U_r = I$. Embed in a doubly coprime factorization
$$\begin{pmatrix} V_\ell & U_\ell \\ -N_\ell & M_\ell \end{pmatrix} \begin{pmatrix} M_r & -U_r \\ N_r & V_r \end{pmatrix} = \begin{pmatrix} I & U_\ell V_r - V_\ell U_r \\ 0 & I \end{pmatrix},$$
- Show the existence of a unique, stabilizing, controller for which the **characteristic function** R_L, defined by
$$R_L := U_r^* M_r - V_r^* N_r = M_\ell U_\ell^* - N_\ell V_\ell^*$$
is in H_+^∞ and has the DSS factorization over H_-^∞, given by
$$R_L = \Phi_J^* S_J = S_K \Phi_K^*.$$
For more on characteristic functions, see [5]
- Show that
$$\begin{aligned} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix} &:= \begin{pmatrix} -N_\ell^* \\ M_\ell^* \end{pmatrix} S_K \\ \begin{pmatrix} K_1 & K_2 \end{pmatrix} &:= S_J \begin{pmatrix} M_r^* & N_r^* \end{pmatrix}, \end{aligned}$$
with $J_i, K_i \in H_+^\infty$.
- Show that
$$\begin{aligned} &\operatorname{Ker} H_{\begin{pmatrix} -N_\ell & M_\ell \end{pmatrix}} = \Omega_J^* H_-^2, \\ &\operatorname{Im} H_{\begin{pmatrix} -N_\ell & M_\ell \end{pmatrix}} = H_+(S_K) = \{S_K H_+^2\}^\perp. \\ &\operatorname{Ker} H_{\begin{pmatrix} M_r \\ N_r \end{pmatrix}} = S_J^* H_-^2, \\ &\operatorname{Im} H_{\begin{pmatrix} M_r \\ N_r \end{pmatrix}} = H_+(\Omega_K) = \{\Omega_K H_+^2\}^\perp, \end{aligned}$$
where the inner functions are given by
$$\begin{aligned} \Omega_J &= \begin{pmatrix} -N_\ell & M_\ell \\ K_1 & K_2 \end{pmatrix} \\ \Omega_K &= \begin{pmatrix} M_r & J_1 \\ N_r & J_2 \end{pmatrix}. \end{aligned}$$
- Show that all the maps defined in the following table are H_-^∞-isomorphisms with respect to the appropriate H_-^∞-module structures.

Map	Intertw. Relation	Hom
$Z_K : H_+(\Omega_K) \longrightarrow H_+(S_K)$ $Z_K \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = P_+ \begin{pmatrix} -U_r^* & V_r^* \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$	$\begin{pmatrix} \Phi_K^* & I \end{pmatrix} \begin{pmatrix} M_r^* & N_r^* \\ J_1^* & J_2^* \end{pmatrix} = S_K^* \begin{pmatrix} -U_r^* & V_r^* \end{pmatrix}$	H_-^∞
$Z_K^{-1} : H_+(S_K) \longrightarrow H_+(\Omega_K)$ $Z_K^{-1} f = P_+ \begin{pmatrix} -N_\ell^* \\ M_\ell^* \end{pmatrix} f$	$\begin{pmatrix} M_r^* & N_r^* \\ J_1^* & J_2^* \end{pmatrix} \begin{pmatrix} -N_\ell^* \\ M_\ell^* \end{pmatrix} = \begin{pmatrix} 0 \\ I \end{pmatrix} S_K^*$	H_-^∞
$W_J : H_-(S_J^*) \longrightarrow H_-(\Omega_J^*)$ $W_J h = P_{H_-(\Omega_J^*)} \begin{pmatrix} V_\ell^* \\ U_\ell^* \end{pmatrix} h$	$\begin{pmatrix} V_\ell^* \\ U_\ell^* \end{pmatrix} S_J^* = \begin{pmatrix} -N_\ell^* & K_1^* \\ M_\ell^* & K_2^* \end{pmatrix} \begin{pmatrix} -\Phi_J^* \\ I \end{pmatrix}$	H_-^∞
$W_J^{-1} : H_-(\Omega_J^*) \longrightarrow H_-(S_J^*)$ $W_J^{-1} h = P_{H_-(S_J^*)} \begin{pmatrix} M_r^* & N_r^* \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$	$\begin{pmatrix} M_r^* & N_r^* \end{pmatrix} \begin{pmatrix} -N_\ell^* & K_1^* \\ M_\ell^* & K_2^* \end{pmatrix} = S_J^* \begin{pmatrix} 0 & I \end{pmatrix}$	H_-^∞
$H_{\begin{pmatrix} -N_\ell & M_\ell \end{pmatrix}} : H_-(\Omega_J^*) \longrightarrow H_+(S_K)$ $H_{\begin{pmatrix} -N_\ell & M_\ell \end{pmatrix}} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$ $= P_+ \begin{pmatrix} -N_\ell & M_\ell \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$	$\begin{pmatrix} I & 0 \end{pmatrix} \begin{pmatrix} -N_\ell & M_\ell \\ K_1 & K_2 \end{pmatrix} = S_K \begin{pmatrix} J_1^* & J_2^* \end{pmatrix}$	H_-^∞
$H_R : H_-(S_J^*) \longrightarrow H_+(S_K)$ $H_R h = P_+ R h$	$\Phi_J^* S_J = S_K \Phi_K^*$	H_-^∞
$H_{\begin{pmatrix} M_r \\ N_r \end{pmatrix}} : H_-(S_J^*) \longrightarrow H_+(\Omega_K)$ $H_{\begin{pmatrix} M_r \\ N_r \end{pmatrix}} h = P_+ \begin{pmatrix} M_r \\ N_r \end{pmatrix} h$	$\begin{pmatrix} K_1^* \\ K_2^* \end{pmatrix} S_J = \begin{pmatrix} M_r & J_1 \\ N_r & J_2 \end{pmatrix} \begin{pmatrix} I \\ 0 \end{pmatrix}$	H_-^∞
$H : H_-(\Omega_J^*) \longrightarrow H_+(\Omega_K)$ $H = H_{\begin{pmatrix} M_r \\ N_r \end{pmatrix} \begin{pmatrix} M_r^* & N_r^* \end{pmatrix}} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$		H_-^∞

- Explain how all these maps are associated with appropriate optimal control problems. These maps are strongly interrelated through the following commutative diagram.

$$\begin{tikzcd}
H_-(S_J^*) \arrow[rr, "H_{R_L}"] \arrow[d, "W_J"'] \arrow[drr, "H_{\begin{pmatrix} M_r \\ N_r \end{pmatrix}}" near end] & & H_+(S_K) \\
H_-(\Omega_J^*) \arrow[rr, "H_{\begin{pmatrix} -N_\ell^* \\ M_\ell^* \end{pmatrix} \begin{pmatrix} -N_\ell & M_\ell \end{pmatrix}}"'] \arrow[urr, "H_{\begin{pmatrix} -N_\ell & M_\ell \end{pmatrix}}" near start] & & H_+(\Omega_K) \arrow[u, "Z_K"']
\end{tikzcd}$$

- Use this diagram, to obtain related ones by applying the adjoint operation to all maps, or, using doubly coprime factorizations, inverting the maps. For example, problems of robust control turn this way into problems of model reduction. In this connection, see [6]. Most of these connections have not yet been worked out.

As the saying goes "god, (or the devil), lies in the details". It can be easily seen, from glancing at the brief outline, that there is an enormous amount of details needed to tell the full story and that would require a monograph. Whether I can do it myself remains to be seen.

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Monge-Kantorovich optimal transport of matrix-valued distributions

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We overview recent results on generalizations of Wasserstein geometry, originally defined on the space of scalar probability densities, to the space of Hermitian matrices and of matrix-valued distributions and vector-valued distributions on continuous as well as discrete spaces (graphs and networks). We follow a control-theoretic optimization formulation of the Wasserstein-2 metric, having its roots in fluid dynamics (due to Benamou and Brenier). We make contact with the mathematics of quantum mechanics and, in particular, we show that the Lindblad equation of open quantum systems represents gradient flow of the quantum entropy relative to a matricial Wasserstein metric. These result was announced in [1]. At about the same time as [1], closely related approaches showing that the Lindblad equation is a gradient flow, were formulated independently and simultaneously in [2,3]. Generalizations, extentions, and further developments are detailed in [4,5,6,7].

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Green Technology, M-Path Poly-Phase Filter Banks

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The computational workload required to implement a digital signal processing filter is proportional to the ratio of sample rate to spectral transition bandwidth. Large ratios indicate poor condition number for the computations and make the implementation costs prohibitive. Often, small transition bandwidth is associated with narrow passband bandwidth which means we also have a large ratio of sample rate to filter bandwidth and the sample rate far exceeds the required Nyquist rate for the filtering process. We improve condition number and reduce implementation cost by a sequence of three processing tasks. We first perform an input signal conditioning operation with an $M-$path filter that performs $M-$to-1 bandwidth and sample rate reduction. This is followed by a filter that the implements the desired filter specifications at the reduced sample rate. This filter operating at $\frac{1}{M}$-th sample rate has $\frac{1}{M}$-th of the number of taps and is performed at $\frac{1}{M}$-th sample rate for a workload reduction on $\frac{1}{M2}$. The filtered signal is then processed by an M-path filter that performs the dual task of $1-$to-M up-sampling while preserving the reduced bandwidth. The cost of the $M-$to-1 down sampling and the $1-$to-M up sampling filters is shown to be surprisingly small. The total workload of the three cascade filter chain is usually an order of magnitude below that of the original direct implementation. There are filtering tasks for which large ratio of sample rate to transition bandwidth does not correspond to a commensurately large ratio of sample rate to passband bandwidth and the sample rate reduction appears not to be a viable method to improve the algorithm condition number. This paper describes a technique that uses Mpath perfect reconstruction (PR) non-maximally decimated filter banks (NMDFBs) to form multiple narrow bandwidth filters that span the sample rate spectral span. The cascade resampling operation is applied to each narrowband sub-channel which supports the desired significant workload reduction and condition number improvement. The multiple base-banded narrowband signals are recombined in an M-path synthesis filter. Here again we find the computational costs to perform the M-path $\frac{M}{2}-$to-1 and $M-$path $1-$to-$\frac{M}{2}$ analysis and synthesis filter banks represents a small increase relative to the single channel cascade. The first benefit of this process is that the computational workload of the cascade analysis-synthesis filter bank filter implementation is typically an order of magnitude below that of the tapped delay line, direct implementation, of the same filter. A second benefit is that a high data rate input time series is partitioned into a set of multiple, reduced sample rate, intermediate time series processed by reduced speed parallel arithmetic processors. This process enables simple, reduced cost, processing of input signals with GHz sample rates.

The Scope of Semidefinite Programming

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One of the main developments in optimization over the last 15 years is Semi-Definite Programming. It treats problems which can be expressed in terms of Linear Matrix Inequalities (LMIs). Any such problem is necessarily convex, so determining the scope and range of applicability comes down to the question:

How much more restricted are LMIs than Convex Matrix Inequalities?

Two different classes of problems naturally arise. The first considers inequalities in which the coefficients are matrices, and the unknowns are real variables (vectors in $\mathbb{R}^n$). Studying such inequalities requires a mixture of matricial functional analysis, algebraic geometry and real algebraic geometry. Much of the recent progress has been due to Claus Scheiderer.

Another class of problems involves inequalities in which the unknowns are also matrices. In this situation, the dimension of the matrices is generally not specified or relevant. Such problems arise in linear systems engineering. These dimension free problems lead directly to a new area, which might be called free real algebraic geometry.

Owing to new developments, we now have some understanding of the scope of LMI methods. The talk will describe the current situation.

Operator algebras and signal processing

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A fundamental tool in multi-band signal processing is the design of filters which serve to break up an input-signal, speech or image, along a prescribed set of frequency-bands (for example low-pass and high-pass). The input signal itself is a discrete-time signal, and so the frequency variable is complex, the frequency-response. A main requirement is that separate frequency components be uncorrelated (so orthogonal with reference to a Hilbert space.) That is required for transmission, and the receiver's output signal is then merged from its band components. As it turns out, the construction of such filters amounts to realizations of representations of a certain algebra of operators O_n, generated by a finite number n of isometries, one for each band, having mutually orthogonal ranges, and summing to the identity. Hence the output is a faithful version of the input-signal.

The early work on O_n was initiated by J. Cuntz in the 1970s, and the algebra O_n is often named after him. (At the time, no one expected any connections to signal processing at all.) While representations of the O_n algebras are of interest in their own right, and enter into the analysis of systems of non-commuting operators; the subject has by now proved to be of independent interest. Because of recent joint work with Alpay and Lewkowicz, we shall focus here on representations that arise from this application to sub-band filters in signal processing. In the talk, we outline an account of joint results on the use of representations of the O_n-relations arising in these filter problems; and it even includes applications to the study of fractals, and geometric measure theory.

The versatility is not surprising since Cuntz algebras, as C^*-algebras are infinite, and by their nature, their representations reflect intrinsic self-similarity, characterizing the problem at hand. Thus the representations serve to encode sub-bands, and more generally iterated function systems (IFSs), their dynamics, and their self-similar measures. At the same time of Cuntz? paper, the O_n -representations offered a new non-commutative harmonic analysis. Although the Cuntz-algebras initially entered into the study of operator-algebras and physics, it is only in recent years, the study of their representation, has blossomed, and it has found increasing use in a multitude of applied problems, including wavelets, fractals, and signals.

Compressed sensing in Quaternionic analysis

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We study the problem of compressed sensing for quaternionic Fourier matrices as arising in color representation of images. We will show that such matrices are allowing a sparse reconstruction by means of an l1-minimization with high probability. We will give explicit expressions for this probability and show how this method can be used for other discrete Fourier transforms. Examples of sparse sampling of color images are provided.

Positive Realness of Descriptor Fractional Systems: Test and Output Feedback

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We present an algebraic method for testing the real-positiveness of time-invariant descriptor fractional linear systems. If a defect of real-positiveness is found, we show how a proportional output feedback can be used to make the system real-positive. We find the maximal open set in which every value of the proportionality factor makes the system real-positive by output feedback.

Our approach relies almost exclusively on symbolic computations and thus avoids any instability that can appears in numerical algorithms. Real-positiveness is shown to be equivalent to the emptiness of the some real algebraic sets. Similarly finding the output feedback that makes the system real-positive is shown to be equivalent to determine when some parametric real algebraic sets are empty as a function of the parameter.

From Brownian motion to bits (and back)
The distortion-rate function of sampled Wiener processes

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Is it possible to encode a path of the Wiener process using a finite number of bits per time lag and recover it with a bounded mean squared error (MSE) distortion? if yes, what is the minimal MSE that can be attained subject to the bitrate constraint? The answers to these two question was given in Berger's PhD thesis, that shows that the optimal tradeoff between code bitrate and distortion is described by Shannon's distortion-rate function (DRF) of the Wiener process. That is, his result implies that for any positive bitrate, there exists a coding scheme that allows recovering of the path of the Wiener process with MSE equals to Shannon's distortion-rate function. Berger's result, however, does not take into account practical considerations in encoding analog process. Indeed, hardware and other implementation constraints restrict access only to samples of the continuous-time path, taken at some finite time resolution.

In this talk we consider the minimal MSE in recovering the path of the Wiener process, but from a code that is only a function of samples of this path. In particular, we derive a distortion function that depends both on the code bitrate and the sampling rate. For example, this function implies that for any bitrate, distortion less than 1.12 times Shannon's DRF can be attained by providing only a single bit per sample of the path. Our results imply that the optimal encoding strategy is obtained by two steps: first interpolate the continuous-time waveform from its uniform samples, and then encode the result of this interpolation in an optimal manner. Since this interpolation before encoding is unfeasible in some scenarios, we also consider the case in which interpolation is performed only at the decoder. Namely, the discrete-time samples are encoded using a source code which is optimal with respect to the discrete-time samples. We show that there is a positive gap between this sub-optimal scheme and the optimal scheme, although the ratio of the two distortions is bounded by 1.28. That is, ignoring the continuous-time origin of the samples, but otherwise encoding them in an optimal manner, increases distortion by up to this factor.

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Inverse source problems in magnetostatics: linear estimator for moment recovery

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We will discuss some inverse problems for Laplace-Poisson partial differential equations (PDE) with source term in divergence form, in dimension 3. We consider situations where incomplete (noisy) Cauchy data are given in some restricted region of the space (accessible to measurements) from which the unknown source term is to be recovered, at least partly.

These issues arise in many physical problems related to non-destructive inspection, in particular for electromagnetic phenomenon modelled by Maxwell's equations, under quasi-static assumptions. They are ill-posed inverse problems, that need to be regularized in order to be constructively solved.

We will more specifically consider related problems from planetary sciences and paleomagnetism, concerning magnetization recovery from magnetic data [1, 2]. There, the magnetization distribution supported in thin rocks samples is to be estimated from measured values of the normal component of the (weak) magnetic field, measured by a very sensitive magnetometer (SQUID, Superconducting QUantum Interference Device). The magnetization is therefore assumed to have a rectangular (horizontal) support, while the normal magnetic field is measured on a parallel rectangle located above. They are related together by means of convolution operators with truncated Poisson and Riesz kernels, the components of the magnetic field being harmonic in the half-space located above the magnetization support (the sample). We first tackle the issue of estimating the net moment of the magnetization (its mean value, a vector in $\mathbb{R}^3$), an important preliminary step towards the full inversion problem. Observe that both are ill-posed, in that the moment recovery problem lacks stability, while the magnetization recovery issue itself suffers from non-uniqueness of its solution (silent sources, that fortunately possess vanishing moment). Note also that solving the moment recovery problem not only provides an *a priori* estimate of the mean value of the unknown magnetization, but also an appropriate direction.

We will show how do harmonic analysis tools, together with approximation techniques allow to set assumptions for well-posedness (stability) and to constructively solve for the above moment estimation issue. This is done by building a set of functions against which the scalar product of the available values of the normal magnetic field (taken on the data set) best quadratically approximates the components of the magnetic moment, under some norm constraint. Resolution algorithms and numerical illustrations will be provided. We will also discuss the links with Hardy spaces of gradients of harmonic functions in the upper half-space.

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A remark on sliding mode differentiation and calculus of variation

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This talk is devoted to the following remark: the 2-sliding mode [1] differentiation approach can be recast in the framework of calculus of variation. More precisely, the Levant's differentiator [2] is expressed via the Euler-Lagrange equation, as the stationary point of a functional representing a regularized cost function, with L_2 regularization. Connections with optimal control thus follow since the Euler-Lagrange equation is a particular instance of the Pontryagin maximum principle which in turn is the basic foundation of optimal control (see e.g. [3]).

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A new approach to solution of completely integrable PDEs

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A theory of vessels, developed by M. Livšic, can be considered as a generalization of the theory of nodes, as it has been shown in [3]. We are going to discuss an application of this theory to solutions of completely integrable PDEs, demonstrating how the basic ingredients of the classical (inverse) scattering theory arise in the setting of vessels. For this task, we will use the example of Korteweg-de Vries (KdV) equation. At this moment it is known that a simplest, two dimensional vessel is equivalent to a generalization of either KdV or of NonLinear Shrödinger (NLS) equation [2]. This result will be explained, providing tools for further demonstration of the application of the theory of vessels to solutions of KdV: 1. the classical Gelfand-Levitan theory has been recently incorporated into the theory [1], 2. vessels approach enables to deal with analytic parameters, and the first steps in this direction were shown in [3], 3. soliton solutions based on finite dimensional realization of the system theory were presented in [4]. Notice that the apparatus of the operator/system theory is applicable to the mentioned PDEs, as it was demonstrated in a joint work with D. Alpay and V. Vinnikov [5].

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On Parametrization of All the Exact Pole-Assignment State-Feedbacks for LTI Systems

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State-feedback is one of the most important notions of control systems. It appears as a solution of the LQR problem and in many more optimal control problems. On the other hand, many system performance objectives, are given in terms of pole-assignment. Occasionally, these requirements are conflicting. For example, LQR state-feedbacks tend to locate the closed-loop eigenvalues on the left half-plane near the imaginary axes, where we might endanger the system stability and might have unsatisfactory response indices.

One of the attitudes to solve this problem is by parametrizing all the pole-assignment state-feedbacks of the given system, where the exact pole-locations are given parametrically. In this way we get two sets of parameters: one is related to the given exact pole-locations and the other is a set of free parameters obtained from the pole-assignment state-feedbacks parametrization. The next step is to apply the optimization requirements to the free parameters in the state-feedbacks parametrization, while leaving the closed-loop poles parameters unchanged. In this way we give the designer the freeness to determine the final location of the closed-loop poles, according to the desired closed-loop system response, while guaranteing the system optimality (relative to the final placement of the poles).

In order to carry out this program, we first need to find a parametrization of all the pole-assignment state-feedbacks for a given controllable system. In the talk, such a parametrization will be introduced, under the assumption that the set of poles to be assigned (parametrically) contains sufficient pure-real poles, where "sufficient" means at least the number of parity alternations in a sequence of the sizes of subsystems, generated from the given system. The parametrization is also proved to be complete, under the above mentioned assumption, in the sense that any state-feedback assigning the given set of poles to the closed-loop system, is included in the parametrization. The talk is based on [1] . The parametrization is based on a recursive substructure of controllable systems, that was first introduced in [2] . Parametrizations of all the pole-assignment state-feedbacks (up to a set of measure 0), under different assumptions, can be found in [3]and [4].

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Selfajoint Vessels and Impedanmce-Conservative Overdetermined Multidimensional Systems

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A $1D$ system is said to be scattering-conservative or impedance-conservative if the energy balance

$$\mathfrak{d}\ \langle \mathfrak{x}, \mathfrak{x}\rangle dt = \langle u, u\rangle - \langle y, y\rangle \quad \text{or} \quad \mathfrak{d}\ \langle \mathfrak{x}, \mathfrak{x}\rangle dt = 2\mathfrak{Re}\ \langle u, y\rangle$$

holds, respectively. It is a well known fact that transfer functions of scattering-conservative systems and impedance-conservative systems are analytic contractive functions on the unit disk and functions with positive real part on the upper half plane, respectively [1]. The interplay between these functions is done via the Möbius transform, and on the systems level, the interplay is by the diagonal transformation of the system variables [4]. In the case of impedance-conservative systems, two functional models are considered: Reproducing kernel Hilbert spaces with the reproducing kernels $\frac{S(z)+S(w)^*}{-i(z-\overline{w})}$ and a $L^2(d\mu)$ space where the measure $d\mu$ is given by the Herglotz integral representation [2].

In this talk, we consider these phenomena and relations in the setting of $2D$ overdetermined systems (or equivalently, commutative two-operator vessels). The $2D$ scattering-conservative systems or equivalently the corresponding quasi–hermitian vessels are well studied, see for instance [3]. We here present the $2D$ impedance-conservative systems and we introduce a new notion, *the selfadjoint vessel*. The interplay between these two types of systems is done by the generalized diagonal transform and the joint characteristic functions are related by the Möbius transformation. In the case of selfadjoint vessels, two functional models arise. The first is a reproducing kernel Hilbert space of analytic sections defined on a real compact Riemann surface X with reproducing kernel

$$T(q)K_{\zeta}(q,r) + K_{\zeta^{\tau\vee}}(q,r)T(r)^*,$$

where $K_{\zeta}(q,r)$ is the Cauchy kernel corresponding to $\zeta \in J(X)$. The second, is an $L^2(d\mu)$ space of sections of a vector bundle over the fixed points (with respect to the involution τ) of X. The measure $d\mu$ is given by the Herglotz integral representation version for real compact Riemann surfaces.

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Realizations of Non-Commutative Rational Functions Around a Matrix Centre: A Generalization of the Fliess-Kronecker Theorem

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This talk focuses on the theory of realizations of rational non-commutative (nc) functions which are regular at a non-scalar matrix point $Y = (Y_1, ..., Y_d) \in (\mathbb{C}^{s\times s})^d$, of the form

$$\mathfrak{r}(X_1, ..., X_d) = I_m \otimes D + (I_m \otimes C)\left(I_{Lm} - \sum_{i=1}^{d} \mathbf{A}_i(X_i - I_m \otimes Y_i)\right)^{-1}\left(\sum_{i=1}^{d} \mathbf{B}_i(X_i - I_m \otimes Y_i)\right) \quad (4)$$

where $\mathbf{A}_1, ..., \mathbf{A}_d : \mathbb{C}^{s\times s} \to \mathbb{C}L \times L$ and $\mathbf{B}_1, ..., \mathbf{B}_d : \mathbb{C}^{s\times s} \to \mathbb{C}^{L\times s}$ are linear mappings, $C \in \mathbb{C}^{s\times L}, D = \mathfrak{r}(Y) \in \mathbb{C}^{s\times s}$ and $X \in (\mathbb{C}^{sm\times sm})^d$. Similarly to the classical theory, I will start by introducing modified definitions for observability, controllability and minimality of realizations of the form above. and proceed by generalizing a singularity theorem from [2] and [3], that is characterizing the domain of regularity of $\mathfrak{r}$ in terms of the linear mappings $\mathbf{A}_1, ..., \mathbf{A}_d$, in the case where the realization is minimal. In [1], the authors proved that the (linear mappings) coefficients $\mathfrak{r}_\omega$ coming from a power series expansion of a nc function

$$\mathfrak{r}(X_1, ..., X_d) = \sum_{\omega} \mathfrak{r}_\omega (X - I_m \otimes Y)^{\odot_s \omega}, \quad (5)$$

must satisfy some compatibility conditions; we found necessary and sufficient new compatibility conditions w.r.t Y on the coefficients $\mathbf{A}_1, ..., \mathbf{A}_d, \mathbf{B}_1, ..., \mathbf{B}_d, C$ and D, for the corresponding sequence of coefficients $(\mathfrak{r}_\omega)$ given by the formulas

$$\mathfrak{r}_\emptyset \equiv D \quad \text{and} \quad \mathfrak{r}_\omega(Z^1, ..., Z^\ell) = C\mathbf{A}_{i_1}(Z^1)...\mathbf{A}_{i_{\ell-1}}(Z^{\ell-1})\mathbf{B}_{i_\ell}(Z^\ell), \quad \text{where } \omega = g_{i_1}...g_{i_\ell}$$

to satisfy the compatibility conditions. Finally, our main result is proving that a nc generalized power series around Y of the form (5) is the power series expansion of a rational nc function $\mathfrak{r}$ around Y if and only if a corresponding (functional model) space built from the coefficients $\mathbf{A}_1, ..., \mathbf{A}_d, \mathbf{B}_1, ..., \mathbf{B}_d, C$ and D is finite dimensional and the new compatibility conditions hold. This is then a generalization of the Fliess-Kronecker theorem for rational nc functions, which tells us that a generalized power series is the power expansion of a rational nc function if and only if the rank of some Hankel matrix is finite.

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Complex Hardy Spaces
Rational Approximation and Applications

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Complex Hardy $\mathcal{H}_2$ spaces, including those of several complex variables (Hardy spaces on tubes) and Clifford variables (conjugate harmonic systems), with scalar or vector, or even matrix-valued on various manifolds, due to their Cauchy structures, are (non-standard) reproducing kernel Hilbert spaces.

The speaker will introduce some rational approximation models with the common feature phrased as adaptive Fourier decomposition (AFD). Such decompositions are violation to the traditional concept of basis. They, however, are highly adaptive to the function to be expanded. Besides fast convergence, they offer, in some important cases, intrinsic decompositions of functions into basic pieces of positive instantaneous frequencies, involving or creating Blaschke products in various contexts.

The results have, so far, applications in signal and image processing and system identification. Such program has been paid attention and joined by a number of renowned international signal analysis groups. In particular, unwinding Fourier expansion independently studied by our and Coifman's groups belongs to the scope of such fast and positive-frequency-decomposition.

The isomorphism problem for noncommutative analytic varieties

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For a noncommutative (nc) subvariety $\mathfrak{V}$ of the nc unit ball, the algebra of bounded analytic functions on $\mathfrak{V}$ — denoted $H^\infty(\mathfrak{V})$ — can be identified as the multiplier algebra of a certain reproducing kernel Hilbert space consisting of nc functions on $\mathfrak{V}$.

In this talk I will show when two such algebras $H^\infty(\mathfrak{V})$ and $H^\infty(\mathfrak{W})$ are isometrically isomorphic (and also completely isometrically isomorphic) in terms of the varieties $\mathfrak{V}$ and $\mathfrak{W}$. We will also focus in the homogeneous case in which we were able to obtain some sharper results. In addition, we will discuss the algebras of bounded analytic functions that extend continuously to the boundary of the nc ball.

Along the way I will present a nc version of the Nullstellensatz for both the homogeneous as well as the commutative case.

The talk is based on a joint work with Eli Shamovich and Orr Shalit.

On quaternionic metaharmonic layer potentials in $\mathbb{R}^2$

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In this talk we give overview of the Hilbert formulas on the unit circle for α-hyperholomorphic function theory. We present several boundary value properties for the 2D quaternionic metaharmonic layer potentials.

Talk based on joint works with J. Bory Reyes, R. Abreu Blaya, M. A. Pérez-de la Rosa.

Plane waves decomposition, hypergeometric functions and Twistors

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We present some remarks concerning Whittaker's, Bateman's and John's formula of integral geometry and their links with Klein correspondance. We apply these remarks, in the context of twistor theory, to the hypergeometric differential equations and to the operator

$$\Delta_3 = \frac{\partial^3}{\partial x^3} + \frac{\partial^3}{\partial y^3} + \frac{\partial^3}{\partial z^3} - 3\frac{\partial}{\partial x}\frac{\partial}{\partial y}\frac{\partial}{\partial z}.$$

This is joint work with Oumar Wone.

Ternary algebras and applications

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We introduce of a class of regular functions defined on three dimensional real and complex hypercomplex algebras. The analytic theory is build on the basis of conjugations. We provide the solution to the Gleason's problem which gives rise to Fueter–type variables and we conclude with several applications in physics.

Multicomplex Analysis and a Multicomplex Fourier Transform

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The past decade has seen a resurgence of interest in spaces of commuting complex units, their algebra, analysis, especially a theory of holomorphic functions in this sense. This talk gives an overview of analysis on nested multicomplex spaces with a view to defining a Fourier Transform in this direction.

Noncommutative hyperbolic metrics

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We define a pseudometric on noncommutative domains that possesses a noncommutative Schwarz–Pick property: every noncommutative function is a contraction. The pseudometric is defined in purely geometric terms and can be calculated analytically for domains defined by a noncommutative hermitian kernel, in particular for "generalized balls" that appear naturally in the study of interpolation problems and that include all matrix convex sets. We show that under natural conditions (the noncommutative hyperbolic metric is nondegenerate and blows up as we approach the boundary), two noncommutative domains admit a noncommutative bijection iff they are isometric. This is talk is based on a joint work (in progress) with Serban Belinschi.

Algebraic division or interpolation through analytic methods

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In the last 20 years, a lot of methods inspired by analysis reveal to be quite successful towards the solution of effective problems in multivariate polynomial computational geometry. One could refer for example to the realization of Bézout identity thanks to an explicit formula, the construction of "residual currents" attached to holomorphic sections of metrized bundles over $\mathbb{C}^n$ or more generally over a complex analytic varieties in order to realize concretely Grothendieck duality, the deformation of (sometimes difficult to vizualize) complex objects (such as algebraic subvarieties in $\mathbb{C}^n$ or $(\mathbb{C}^*)^n$, rational functions in several variables, Fourier duality) towards much simpler to handle so-called "tropical" combinatorics objects (tropical cycles, tropical polynomials, Legendre-Fenchel duality,...). I will give in this lecture a panorama of such technics, trying to put some of them in situation in front of questions inspired by systems theory (BIBO stability of multi-dimensional filters, classification of multivariate Mellin transforms of $1/F$ when $F \in \mathbb{C}[X_1^{\pm 1}, ..., X_n^{\pm}]$, etc.). I listed here in chronological order some of the references which where my guidelines during the preparation of this talk.

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