Social Groups, Social Media, and Higher Dimensional Social Structures: A Simplicial Model of Social Aggregation for Computational Communication Research

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Social Groups, Social Media, and Higher Dimensional Social Structures: A Simplicial Model of Social Aggregation for Computational Communication Research

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By building on classical communication network literature, we present a computational approach to modeling tightly bound groups and social aggregations as higher dimensional social structures. Using the mathematical theory of simplicial complexes, these groups can be represented by geometric spatial elements (or simplexes) and a social aggregation a collection of simplexes (i.e., a simplicial complex). We discuss the uniting conditions that define a tightly bound group as a higher-dimensional group, which can be mathematically treated as nodes in a network of social aggregation. We utilize Facebook as a particularly relevant example to demonstrate innovative ways researchers can tap into digital data, in addition to traditional self-reported data, to advance communication research using the simplicial model, although the approach is applicable to many questions not involving communication technology.
In September 2009, Lazer and colleagues published an influential article in *Science* about the coming of age of computational social science. They argued that people engage in daily activities on the Internet and leave “digital breadcrumbs,” which “when pulled together, offer increasingly comprehensive pictures of both individuals and groups, with the potential of transforming our understanding of our lives, organizations, and society in a fashion that was barely conceivable just a few years ago” (p. 721). This argument brings new enthusiasm for what Carley (1995) called the field of computational and mathematical organization theory, an approach to the study of organizations using models that are both computational (e.g., simulation, computer-assisted numerical analysis, etc.) and mathematical (e.g., formal logic, matrix algebra, discrete and continuous equations). This argument is also exciting for communication research, because individuals, groups, and organizations are digitally linked in a complex communication network that can be studied by mathematical analyses (Barabási, 2002).

McPhee and Poole (1981) and Poole and Hunter (1984) in the early 1980s, as well as Richards and Barnett (1993) and Fink (1993) in the early 1990s, promoted mathematical modeling in communication research. Computational and mathematical approaches have been pursued by communication researchers on various topics, such as organizational dissolution and disintegration (Tutzauer, 1985), social and organizational networks (Barnett & Rice, 1985; Monge & Contractor, 2003), virtual team alliance (Thomsen, Levitt, & Nass, 2005), public issue priority formation (Zhu, Watt, Snyder, Yan, & Jiang, 1993), terrorist organizations (Corman, 2006), diffusion of innovations (Valente, 1993, 1996), public policy (Woelfel, 1993), discourse in complex adaptive systems (Dooley, Corman, McPhee, & Kuhn, 2003), cultural convergence (Kincaid, Yum, & Woelfel, 1983), intercultural interactions (Armstrong & Bauman, 1993), and international telecommunication networks (Barnett, 2001).

In this article, we build on existing communication network literature, as well as the argument advanced by Lazer and colleagues (2009), to introduce a simplicial model of social aggregation. The term “social aggregation” is used to refer to an organization of individuals and interpersonally bonded clusters of individuals, a community of people and culturally distinct subgroups, or a collection of human nodes and different united social entities that co-exist within a system. In mathematics, the term “simplicial” is used to describe a geometric spatial element (also known as a simplex) with the minimum number of boundary points, such as a one-dimensional line segment, a two-dimensional triangle, a three-dimensional pyramid, etc. (Faridi, 2002; Munkres, 1984). In the proposed model, geometric spatial elements are used to represent interpersonally bonded clusters, culturally distinct subgroups, united social entities, and other inextricably tied social groups in a communication network of social aggregation. In turn, a social aggregation is mathematically represented by a simplicial complex. When applied appropriately,
the simplicial model of social aggregation can be used to generate and test robust communication theories in various subfields through computational modeling and simulation with the wealth of digital network data accumulated on social media and communication technology networks. This article will refer to social media sites as a particularly relevant example of the mathematical concepts described here, although the approach is applicable to many questions not involving communication technology.

In this article, we begin by providing a brief literature review of social network and organizational research to argue for the need to expand network research to higher dimensionalities. Second, we present the conceptual explications of two key constructs: higher-dimensional group and social aggregation. Third, we discuss the methodological advantages of the simplicial model over the classical network approach when considering higher-dimensional groups. Fourth, we introduce the mathematical basis of the simplicial model. Fifth, we offer a series of scenarios to illustrate a simplex as a higher-dimensional group and a simplicial complex as a social aggregation in human communication. Sixth, we briefly highlight the available tools and practical techniques for carrying out a simplicial study. Seventh, we consider the limitations of our current development of the simplicial model and suggest future research directions. Finally, we end with a conclusion and six implications. While we also discuss the model’s implications for several subfields of communication and qualitative methods, this article is aimed at contributing to the conversation on social network analysis and communication network research.

Classical Network Research, Groups, and Organizations

To build on the foundation of communication network literature, we first turn to Rogers and Kincaid (1981) who put forth the following definition of a communication network: “interconnected individuals who are linked by patterned communication flows” (p. 82). Knoke and Yang (2008) defined a social network as “a structure composed of a set of actors, some of whose members are connected by a set of one or more relations” (p. 8). It is important to note the similarities in the definitions, which claim that a network is a social structure shaped by its patterned communication flows and a tie is a relation that interconnects two people. Furthermore, Knoke and Kuklinski (1982) maintained that a network can affect the behaviors, perceptions, and attitudes of the individuals embedded within the network. Therefore, a network is a social structure shaped by its patterned communication flows through the relations among interconnected individuals and the configuration of the network has consequences at the individual and system levels. More recent work advances our understanding of organizational networks as forms that use flexible and dynamic communication linkages connecting multiple organizations and people into new entities (Contractor, Wasserman, & Faust, 2006), such as communities or social systems.

Traditional organizational communication models have been using social network approaches (see, e.g., Farace, Monge, & Russell, 1977; Monge & Contractor, 2001,
to understand interaction within organizations, communities, and social systems for decades. Researchers are often interested in the theory of group formation, group evolution, and communication networks by building models that aid in describing groups as static as well as their often dynamic evolution (see, e.g., Contractor et al., 2006; Monge & Contractor, 2003; Valente, 1995). Scholars have been particularly interested in examining group dynamics and decision-making in organizations, particularly when it comes to the potential to offer tools to act on groups in terms of either strengthening or weakening them (see, e.g., Feeley & Barnett, 1997; Monge & Contractor, 2003; Poole & Holmes, 1995; Sparks, 2008; Valente, 1995).

Moreover, organizational behavior theorists have typically utilized network theory, dynamical systems theory, and the theory of random networks in studying group interaction and organizational behavior, particularly in recent years (see, e.g., Contractor et al., 2006; Perlow, Gittell, & Katz, 2004; Stohl & Stohl, 2007). Network analysts have developed powerful tools to study the dynamic evolution of social networks using the concept of exponential graphs (Snijders, 2002), which consider a sequence of possible configurations of a network (a Markov chain) and use an exponential function to model the probability that at a given instant the network presents some particular features, like the number of lines, the number of triangles, the number of “two-stars” (triangles with a missing edge), and so on. This recent development, rooted in the work of Erdös and Renyi (1960), has attracted much research interest.

For decades, network researchers have consistently found that variability in communication patterns among group members affects performance and functioning in organizations and communities (see Bavelas, 1950; Leavitt, 1951). They have paid attention to the complex patterns of group interaction within organizations with a more macroscopic perspective focusing on social structures of the organization and community including the relational patterns of members (see, e.g., Burt, 1992; Feeley & Barnett, 1997; Friedkin, 1998). Common in these approaches reviewed is the recognition that people in organizations, communities, and social systems are connected through communication networks and that these network patterns change over time.

Prior communication network research has given limited attention to the systematic understanding of interpersonally bonded clusters, culturally distinct subgroups, united social entities, and other inextricably tied groups due to the inherent complexities that emerge in group interaction, particularly when members join and leave groups over time, thus dismantling the static snapshots taken at earlier times. The complexities increase when members within groups share distinct bonding conditions (e.g., group identity and ideology, private sphere and worldview) that define them as tightly bound groups, making their in-group interactions different from out-group interactions. We refer to these tightly bound groups as having higher dimensionalities. To extend network research to include such groups, the next section explicates the theoretical constructs of higher-dimensional group and social aggregation.
Conceptual Explications of Higher-Dimensional Groups and Social Aggregation

Classical network literature treats a group as a collection of nodes that are pairwise connected. To illustrate that a higher-dimensional group is more complex than that, let us consider the example of a terrorist cell. A terrorist cell is a unique phenomenon because it is formed of several people, and its members are pairwise connected, but it also assumes the behavior of an entity in itself with its own uniting conditions, and as such, a shared group identity (Sparks, 2005) and ideology (Drake, 1998). A classical communication network model of such a cell would show the first two characteristics but not the last one, because a classical network model is one-dimensional in nature, mainly linking nodes with straight lines.

Furthermore, consider the examples of families, couples, and activist groups to illustrate the construct of higher-dimensional group. Koerner and Fitzpatrick (2002) argued that families can create a conversational climate “in which all members are encouraged to participate in unrestrained interaction about a wide array of topics” (p. 85). Furthermore, a family can also create a conformity orientation in which “family communication stresses a climate of homogeneity of attitudes, values, and beliefs” (p. 85). Therefore, we extend that an open communication climate with an implied sense of trust and a converging climate with a sense of conformity and cohesion are other examples of uniting conditions.

Whitchurch and Dickson (1999) explained that a couple co-creates a “private sphere” and a “private world view” that is unique to the couple (p. 693). These properties concretize over time, leading to the “objectivation of reality,” which bonds the couple as a couple. Sharing a private sphere, a private world view, and the experience of the objectivation of reality are additional examples of uniting conditions. The word private could include a sense of secrecy, especially in the example of a terrorist cell. These emergent properties from a couple also define our conceptualization of a higher-dimensional group, because some scholars have considered dyads in small group research (see Tschan, 2002), although in general, we follow Gastil’s (2009) argument that a small group consists of at least three members.

In a study of two environmental activist groups, Kitchell, Hannan, and Kempton (2000) argued that activist groups use storytelling to generate identity formation and motivate members to take actions and change their own consumption behaviors. The discipline of communication has long been aware of the power of stories and narratives in groups, organizations, and communities, especially in creating a shared history with a coherent past and an anticipated future (Bormann, Cragan, & Shields, 2001; Browning & Morris, 2012; Fisher, 1985). These properties add to our notion of uniting conditions.

In summary, uniting conditions may include, but are not limited to, three subsets of inter-related properties: (a) a private sphere and world view, a sense of shared identity and ideology, and objectivation of reality; (b) an open in-group communication climate with a high degree of trust or a converging climate of conformity and cohesion; and, (c) a shared life story, a narrative history, a coherent past, and an
anticipated future together, and these properties cannot be simply reduced to the dyadic ties that constitute the tight group. In essence, then, uniting conditions are any theoretically relevant properties that create a higher-order group structure that cannot be visualized and analyzed through traditional social network graphs alone. In any given study, uniting conditions derive from the theory employed by the researcher as well as the social context under investigation.

Classical network research has explored how different tie strength levels can be measured by four possible categories of variables: (a) temporal factors, such as amount of time (Granovetter, 1973), communication frequency (Haythornthwaite, 2005), and duration of relationships (Haythornthwaite, 2005); (b) psychological factors, such as emotional intensity (Granovetter, 1973) and intimacy (Granovetter, 1973; Haythornthwaite, 2005); (c) interactional factors, such as reciprocal services and exchanges (Granovetter, 1973, 1983), and multiplexity and variety of communication media (Haythornthwaite, 2001, 2005); and (d) relational tie content, such as transaction relations, kinship relations, etc., with various interests, motives, purpose and drives (Haythornthwaite, 2005; Knoke & Yang, 2008). These variables give different meanings to higher-dimensional groups, because one with high emotional intensity (e.g., a group of close friends) is different from another based on reciprocal services (e.g., certain business relationships). These factors contribute to the necessary uniting conditions that bind clustered individuals into a higher-dimensional group. However, it is the uniting conditions that emerge out of these factors that give the groups higher dimensionalities.

Given the conceptual explication of higher-dimensional groups, we define a social aggregation as a system (or an organization, a community, etc.) made up of a collection of individuals, together with their mutual connections (partnerships, friendships, relations, etc.), as well as the collection of possible higher-dimensional groups within the system. A higher-dimensional group within a social aggregation is a collection of clustered individuals that evolve into inextricably tied entities with distinct uniting conditions. They are interpersonal bonded clusters of individuals, culturally distinct subgroups within a society, and different united social entities that co-exist within a system. Social aggregation and its higher-dimensional groups are important for communication research because, as Poole (1998) argued, the small group should be the fundamental unit of communication research.

To further illustrate the construct of higher-dimensional groups within a social aggregation, an example group of three individuals is useful here. In this group, there are three friends: Armando (A), Bryan (B), and Cho (C). Within the classical theory of communication networks, the possible configurations of such a group are exactly the eight depicted in Figure 1. These configurations correspond to different social structures and are a reflection of different structural tendencies within a society. In a recent article on triads, Faust (2010) discussed 16 configurations based on directed graphs.

To theorize the interaction among A, B, and C beyond pairwise connections, a model needs to take into account the possibility of considering the three individuals as a strongly united group with distinct uniting conditions, which we represent by the full triangle ABC together with its interior shade (see Figure 2).
Analogously, if the network is composed of four individuals, Armando, Bryan, Cho, and Denzel (D), our model makes it possible to introduce four triadic entities and one four-dimensional relation given by the full interior of the solid $ABCD$. Figure 3 presents some examples of essentially different configurations that can be captured with this higher-dimensional approach. A classical communication network approach would not fully distinguish between them.

To further develop this model, the notion of *face* is introduced here. In mathematical language, a full triangle has three one-dimensional faces or 1-simplices (i.e., lines, edges, segments, or sides; they are linkages in classical communication network literature). In turn, every 1-simplex has two faces or 0-simplices (i.e., end points or border points; they are nodes in classical communication network literature). The faces of a 1-simplex are its two end points, the faces of a 2-simplex (a triangle) are its three edges, the faces of a 3-simplex (a pyramid) are its four faces (each of which is a triangle), and so on and so forth. Therefore, the face of an $n$-simplex is always an $(n - 1)$-simplex.

The darker color of the bottom right solid of Figure 3 indicates the interior of the pyramid, which is a three-dimensional object with volume that can be measured mathematically, as opposed to the four two-dimensional faces of each of the other pyramids, which are shaded with a lighter scale of gray. The last picture in Figure 3 (bottom right corner) is thus used to model a higher-dimensional group, more connected than in any of the other cases in which individuals operate and influence each other only via one-on-one interactions (top left pyramid) or via one-on-one combined with triadic relationships. Next, it is important to gain a better understanding.
of how consideration of higher-dimensional groups adds to our existing knowledge of communication networks, which we address in the next section.

Methodological Advantages of the Simplicial Approach

Classical communication network research has laid a strong foundation to study one-dimensional structures based on the existence (or lack) of connections between different individuals (see, e.g., Granovetter, 1973, 1985; Rogers & Kincaid, 1981). Formal models of network analysis, such as graph theory, have been used for this endeavor in the past. In their suggestion to advance communication network research, Contractor et al. (2006) proposed the need to overcome the limitations imposed by dyads (pairwise or one-on-one connections). Quoting Jones et al. (1997), they stated that “ignoring the larger structural context, these studies cannot show adequately how the network structure influences exchanges” (p. 684). At this point, it is important to discuss how the simplicial approach differs from, and provides methodological advantages over, classical methods when considering higher-dimensional groups.

First, although the idea of using simplexes to describe and model social interactions and connections is not totally new, as it was developed in Q-analysis theory (Atkin, 1972, 1974, 1977) and advanced in more recent theoretical work (see Jacobson & Yan, 1998; Legrand, 2002), there are some fundamental differences between our proposed approach and, for instance, that of Atkin, who used simplicial complexes to represent (binary) relations in a Cartesian product of two sets. Simplicial complexes in our model indicate actual social entities, whereas relations (binary and n-ary alike) that comprise them are, literally, their faces. Just as a human

![Figure 3](image-url) Higher-Dimensional Configurations for a Network of Four Individuals.
body is more than a mere aggregate of its organs, social groups are more than their constituents.

Second, the recent approach of exponential random graph models (ERGM) (Shumate & Palazzolo, 2010; Su, Huang, & Contractor, 2010) pushes the classical analysis further away from simply dyadic relations. However, an ERGM would not be able to find different representations for the pyramids in Figure 3. They would all be represented as a graph, like the one in the upper left corner of Figure 3. Therefore, it still only relates to one-dimensional structures; thus, it is not fully capable of modeling social aggregation and its higher-dimensional groups.

Third, it is important to note another approach to consider social clusters in network research: block modeling (Doreian, Batagelj, & Ferligoj, 2005). This approach treats individuals with an equivalent structural position within a network as a cluster and the set of ties from one cluster to another as a block, thus reducing a large and complex network into a smaller network that can be interpreted more easily. However, our approach is different from block modeling in that not all members in a social aggregation or higher-dimensional group hold the same structural position. For example, a former member of a tightly bound group may still be connected to an in-group member, but the former member is no longer guided by the group’s current set of uniting conditions, and thus the former member should not be included in the modeling of the group.

Fourth, a simplicial complex can theoretically be replaced by a colored graph (McMorris, Warnow, & Wimer, 1993). For example, a two-dimensional simplex (i.e., a full triangle) can be replaced by a graph with three green vertices (connected by three edges, similar to a triangle in traditional network research) and a red vertex (representing the two-dimensional face). In a colored graph, this red vertex is also connected to each of the three green vertices. The graph obtained in this way now has four vertices (instead of three as in a simplicial complex) and six edges (instead of three in a simplicial complex). The situation grows in complexity as the dimension increases. For instance, a three-dimensional simplex (i.e., a pyramid) can be represented by four green vertices (the original vertices), four red vertices (the two-dimensional faces, which are full triangles), and a blue vertex (the three-dimensional face, the pyramid). The number of edges connecting the green, red, and blue vertices now totals 26 (i.e., $6 + 12 + 8 = 26$). Thus, while in principle it is always possible to reduce the representation to a problem involving only graphs, the resulting objects are inherently more complex than the simplicial complexes that they replace.

In the discipline of communication, Barnett and Rice (1985) maintained that multidimensional scaling applied to social networks has not been satisfactory mainly because there has not been an effective way to take imaginary dimensions into account. The mathematical theory of simplicial complexes introduced next allows us to build on the existing understanding of communication networks by providing a richer graphical representation of social interactions within groups, organizations, and communities, and it determines quantitative mechanisms to describe the robustness of a social structure.
Mathematical Basis of the Simplicial Model of Social Aggregation

To describe the complex phenomenon of social aggregation as explicated, a network model, one that allows for higher-dimensional groups in which three or more people give rise to something more than a set of dyadic conversations, is needed. Therefore, the article turns to the mathematical theory of simplicial complexes (see Faridi, 2002; Munkres, 1984). The use of a simplicial approach for social science research was proposed by Mannucci, Sparks, and Struppa (2006), and has been recently employed by Ren and colleagues (2011) to model wireless communication networks.

The construct of a simplex is a generalization of the higher dimensions of the intuitive notions of triangles and pyramids. A zero-dimensional simplex is a point $P$. A one-dimensional simplex is a link (or an edge), including its end points. If $P$ and $Q$ are the end points, we denote this 1-simplex by $[P, Q]$. In a given collection of individuals, when some (or all) of the nodes are connected, a structure that is one-dimensional (though it may still have zero-dimensional components) is obtained. A two-dimensional simplex, or 2-simplex, is the inside of an equilateral triangle $[P, Q, R]$ whose vertices are the points $P, Q$, and $R$, which represent three individuals in our model. The triangle itself (meaning the three edges constituting its perimeter, as well as its three vertices) is part of the 2-simplex. A 3-simplex (or a three-dimensional simplex) $[P, Q, R, S]$ is a tetrahedron or the pyramid with four faces, each of which is an equilateral triangle including its four two-dimensional faces, as well as its six edges and four vertices.

To construct a four-dimensional simplex, or 4-simplex, one starts with a regular tetrahedron $[P, Q, R, S]$ (the 3-simplex), and then one takes a point $T$ in the fourth dimension, such that the distance between $T$ and each of the points $P, Q, R$, and $S$ is the same. One then fills up the object $[P, Q, R, S, T]$ in the four space by taking all the points in the four space that are “inside” or “on” the boundary created by the points $P, Q, R, S,$ and $T$ (in the same way in which a triangle is the set of points “inside” three given points $P, Q$, and $R$ or along the three edges delimited by the vertices). The resulting object is a 4-simplex. The same steps can be repeated to create higher dimensional simplexes (see also May, 1993).

The objects of ultimate interest in this article, however, are not simplexes, but rather simplicial complexes. To define what a simplicial complex is, the notion of the “face” of a simplex needs to be reintroduced. As previously explained, the face of an $n$-simplex is always an $(n-1)$-simplex. Therefore, the faces of a 3-simplex (a pyramid) are its four faces (each of which is a triangle), the faces of a 4-simplex (a pentatope) $[P, Q, R, S, T]$ are the five 3-simplexes that one obtains from the pentatope by ignoring one vertex at a time (i.e., they are $[P, Q, R, S]$, $[P, Q, R, T]$, $[P, Q, S, T]$, $[P, R, S, T]$, and $[Q, R, S, T]$), etc. Given a simplex, one can consider its faces, and then the faces of its faces, and so on. This leads to the notion of an $m$-face of a simplex. Simply put, given an $n$-simplex $S = [P_0, P_1, \ldots, P_n]$, point $P_i$ is the 0-face of $S$, and the 1-simplexes $[P_i, P_j]$ are the 1-faces, the 2-simplexes $[P_k, P_i, P_j]$ are the 2-faces, etc.

Therefore, a simplicial complex can be defined as the union of several simplexes (possibly of different dimensions, e.g., 0-simplexes, 1-simplexes, 2-simplexes,
3-simplexes, etc.), including all their faces. In other words, a collection of simplexes, not necessarily linked together by edges or faces, is what is called a simplicial complex. If a face is such that there is no other simplex in the complex containing it, it is called a maximal face (or facet). The key point is that if a particular simplex is part of the complex, like the triangle on the left side of Figure 4, then all its subfaces (its three edges and three vertices in this case) also belong to the complex. In this article, a social aggregation can be mathematically represented by a simplicial complex. As shown in Figure 4, a 0-simplex indicates an isolated individual, a 1-simplex represents two connected individuals, and a 2-simplex denotes a higher-dimensional group of three members. These individuals (i.e., nodes) and groups (i.e., simplexes) exist within a larger organization, community, or system we refer to as a social aggregation.

In classical network literature, Wellman and Berkowitz (1988) explained that a common approach to encode a network is via its adjacency matrix. An adjacency matrix is a collection of numbers $a_{ij}$ such that $a_{ij} = 1$ if the node $i$ is connected to the node $j$ by an edge (indicating a tie between them), and $a_{ij} = 0$ otherwise (this matrix turns out to be symmetric if assumed to be undirected graphs). This article proposes a higher-dimensional extension of the adjacency matrices when multi-index matrices are allowed. In other words, a higher-dimensional group can be treated as a node in a classical communication network. For example, 2-simplexes (triangles) of a simplicial complex can be encoded into a collection of numbers $t_{ijk}$ where $t_{ijk} = 1$ if the nodes $i$, $j$, and $k$ are connected by a full triangle and $t_{ijk} = 0$ otherwise. For three-dimensional simplexes, such as a pyramid, an object with four indices $p_{ijkl}$ can be introduced, assuming a value of 1 when the corresponding pyramid is part of the complex and zero otherwise.

Therefore, in general terms, the number $a_{X_1 X_2 \ldots X_t}$ will be set to be 1 if the $(t-1)$-simplex connecting the nodes $X_1, \ldots, X_t$ is part of the complex and zero otherwise. This collection of multi-index objects, the last one being $a_{X_1 X_2 \ldots X_n}$ where $n$ is the number of nodes of the complex, is called the adjacency vector associated with the complex. Since, given a simplicial complex with $n$ nodes, there can be at most \binom{n}{t} faces consisting of $t$ nodes (i.e., $(t-1)$-simplexes), the total number of possible simplexes in the complex is $\sum_{t=1}^{n} \binom{n}{t} = 2^n - 1$. Centrality notions, more precisely degree centrality and closeness centrality in classical network literature, can naturally be extended to simplicial complexes.

![Figure 4](attachment:image.png)
An important concept in communication networks is degree centrality, which can be understood as the number of links connected to a given node. So the edge-centrality of a node \( A \) can be measured as the number of 1-simplexes that meet at \( A \); this concept is then extended to higher dimensions by saying that the \( d \)-centrality of a node \( A \) is the number of \( d \)-simplexes to which \( A \) belongs. Using the definition of an adjacency vector given above, \( d \)-centrality is given by

\[
K_d(A) = \sum_{1 \leq X_1 < \ldots < A < \ldots < X_d \leq n} a_{X_1 \ldots A \ldots X_d}
\]

Finally, the total centrality can be measured as the sum of all \( d \)-centralities:

\[
K(A) = \sum_{i=1}^{n} K_d(A)
\]

The discussions above indicate how this mathematical model can be used in a dynamic way to represent social aggregations and their changes through time. A social structure can be thought of as being a function \( F \) from the set of positive numbers with values in the space of simplicial complexes. \( F(0) \) represents the initial state of a social structure, and as time \( t \) increases, \( F(t) \) represents the evolution of that social structure. As new relationships are born, the complexity of \( F(t) \) will increase, but at the same time \( F(t) \) can become simpler if some of the members of the social structure leave the structure or if relationships are weakened, as illustrated in Scenario 2 in the next section (this reduces the dimensionality of the simplicial complex, at least in some of its components). Thus, simplicial complexes can be seen as helpful for at least the following three applications: First, simplicial complexes can be used to model and visualize a social structure of higher-order and multiple dimensionalities. Second, computational tools from the theory of simplicial complexes can be used to describe and modify a given social structure of higher-order and multiple dimensionalities. Third, a dynamical theory of simplicial complexes can be used to simulate the growth and structure of a given higher-order and multiple dimensional organization, extending classical Monte Carlo methods (a class of algorithms in computational science that uses repeated random sampling; see Kalos & Whitlock, 2008) for exponential graphs to exponential simplicial complexes.

In the following section, a series of scenarios is offered to explain why higher dimensionalities are needed to model social aggregation. What follows are simply examples to show how simplicial complexes can be used to visualize such social aggregations.

Illustrative Scenarios of Simplex and Simplicial Complex in Human Communication

In the physical and life sciences, it is increasingly common for researchers to conduct e-science (Schroeder & Fry, 2007) using large-scale scientific data accumulated using
cyberinfrastructure, also known as distributed and networked infrastructure (Atkins et al., 2003; Kee, Cradduck, Blodgett, & Olwan, 2011). In the series of scenarios below, connections on social media and social network sites (boyd & Ellison, 2007) are assumed to be the online reflection of people’s offline social relationships. Because social media, such as Facebook, reflect a user’s social capital (Ellison, Steinfield, & Lampe, 2007; Valenzuela, Park, & Kee, 2009), the example of Facebook is used below to show that communication researchers can tap into Facebook data to advance computational social science using the simplicial model proposed in this article. Future research can utilize actual digital data in addition to traditional self-reported data.

**Scenario 1**

A higher-dimensional group is composed of three college students: Derek, Maya, and Christos. They make up a strong social group when they frequently come together and socialize as a unit among other college students on campus. Over time, their communication activities generate uniting conditions. The simplex that describes this social group is a 2-simplex. The nodes are $D$, $M$, and $C$ (Derek, Maya, and Christos); the 1-faces describe the interaction between any two of them ($D$ and $M$, $D$ and $C$, etc.), and the entire simplex describes the interaction of the three components as a tight social group.

Assume that they are also friends on Facebook. A higher-dimensional group with uniting conditions can be detected by the following activities: First, the expression of a private sphere and world view, a shared identity and ideology, and objectivation of reality can be seen in overlapping profile information, joining and participating in overlapping Facebook groups, endorsing (e.g., clicking the “Like” button) the same sets of postings, using similar language, code words, linguistic styles, etc. Second, the presence of an open communication climate or a convergence climate can be demonstrated by members interacting with each other on Facebook via multiple channels, including publicly commenting on each other’s status updates and postings, sending each other private individual and group messages, chatting in real time, etc. Furthermore, we may also observe the use of emoticons to show emotional intimacy and intensity. Third, a shared life story, narrative history, a coherent past, and an anticipated future together could be documented by tagging each other in common pictures, videos, and postings, and indicating the intention to attend common offline events organized and publicized on Facebook, etc.

The graphical representation of their offline connections and higher dimensionality can easily be obtained by actual Facebook data as described above, and this relationship representation is indicated in Figure 5. This simplicial complex contains only one maximal face, namely the 2-simplex.

**Scenario 2**

Suppose now the social group breaks down as the consequence of a conflict between Derek and Maya. The new structure is a one-dimensional simplicial complex. It is
made of three nodes (as before) and three 1-simplexes (the relationship between Maya and Christos, the relationship between Derek and Christos, and an awkward relationship between Maya and Derek, “awkward” because they stopped interacting with each other on Facebook in the manner described in Scenario 1 although they remain as Facebook friends). What is now missing is the 2-simplex, because the three components do not come together as an intact social group any more, no longer fully defined by their distinct uniting conditions. The maximal faces are now the 1-simplexes shown in Figure 6.

Scenario 3

A new semester begins and Maya and Christos continue to socialize, but now very little or without Derek (note that Christos and Derek are still friends). In a class, Maya and Christos meet a new friend, Steven (S), and they form a team for a class project. Steven becomes a Facebook friend with Maya and Christos. They now socialize as a new united social group \([M, C, S]\), which functions as a single unit in their class and campus life. The representation now is a simplicial complex in which we have simplexes of different dimensions. There are four nodes \([D, M, C, S]\), and two triangles. One is the triangle representing the original (now weakened) social group involving nodes \([D, M, C]\). This is a one-dimensional triangle, as only the linkages exist. The second triangle represents the new social group involving nodes \([M, C, S]\). Because this is a united social group, the representation is actually a 2-simplex \([M, C, S]\), to which two one-dimensional simplexes \([M, D]\) and \([C, D]\) are attached to. These two 1-simplexes are in turn attached on the 0-simplex \(D\). See Figure 7 for Scenario 3. The maximal faces are \([M, D]\), \([C, D]\), and \([M, S, C]\). Note that nodes \(S\) and \(D\) are not connected by a link.

Scenario 4

Building on Scenario 3, the group \([M, C, S]\) has a new member, Belinda (B). The new member is fully integrated into the group, and Belinda becomes a Facebook friend

Figure 6  Modeling the Derek–Maya–Christos Social Group on Facebook with three 1-Simplexes.
with Maya, Christos, and Steven. The group now must be represented as a 3-simplex \([M, C, S, B]\), which is therefore also a maximal face. On this simplex with four faces, each of which describes the interaction of the subgroup that exists when one of the components is absent (when Steven is away attending a class, \([M, C, B]\) still socialize as a smaller social group; similarly, when Christos is away at lunch with his friend Derek, \([M, S, B]\) are a fully functioning social group; etc.). The new members, Steven and Belinda, and the original member, Derek, are not connected on Facebook. So the simplicial complex to represent this more complex social relationship is now made of a tetrahedron (representing \([M, C, S, B]\)) and two 1-simplices (the relations \([M, D]\), and \([C, D]\)) plus, as the definition requires, all their subfaces (see Figure 8 below).

**Scenario 5**

Finally, a third new member, Adele (A), joins in the group. Moreover, A and her new friends, S, M, C, and B form a strongly united group. The representation would need to be represented by a 4-simplex \([M, C, S, B, A]\) to describe the new social group, plus the old linkages \([M, D]\) and \([C, D]\). Since the 4-simplex \([M, C, S, B, A]\) is a four-dimensional geometrical object, it is impossible to represent it on paper in a way that suggests the full complexity of the situation. Some of the three-dimensional faces of the 4-simplex could be drawn, but they would only be a partial representation of the simplicial complex. Van de Ven and Poole (1995) argued to study the sequences of events that unfold in changes in groups and organizations. These five scenarios represent these sequences of events and show how higher-dimensional simplexes and simplicial complexes can model increasingly complex social structures.
Practical Tools and Software Packages

This article focuses on the conceptual explication for and mathematical basis of the simplicial model of social aggregation. Although page limitations do not allow for a full discussion on tools and techniques, we would like to provide an overview of the options available for future research. Matsuo, Mori, and Ishizuka (2008) reviewed several tools that can be used for social network mining on the Web, including Referral Web (Kautz, Selman, & Shah, 1997), a social network extraction system that can be used to mine e-mail archives, Flink (Mika, 2005), an extraction system that can be used on Web pages and e-mail messages, and Polyphonet (Matsuo et al., 2007), a mining system that recognizes different types of relations on the Web and social network sites.

Matsuo and colleagues (2008) provided examples of pseudocodes and modules as well as basic and advanced algorithms for how to perform online data mining. Mika (2005) explained that tools like Flink can extract online data for social network analysis. After successful extraction, one can consider using an open-source computation tool called Polymake (http://www.polymake.org) to visualize and calculate measures of simplicial complexes discussed in this article. Gawrilow and Joswig (2000) explain the design of the Polymake software tool and provide tutorial for basic and advanced applications.

In our program of research, we worked with several mathematicians and computational scientists to perform a simulation based on the simplicial model, using a computer program called CoCoA. CoCoA (Computations in Commutative Algebra, http://cocoa.dima.unige.it/) is an open-source computer algebra system that can be used to compute with numbers and polynomials. The CoCoA Library is available under GNU General Public License. It has been ported to many operating systems, including Macintosh on PPC and x86, Linux on x86, x86-64, & PPC, Sun Solaris on SPARC, and Windows on x86. The CoCoA Library is mainly used by researchers but can also be useful for simple computations. Using the CoCoA program, our research team performed a computational simulation to model the diffusion of a health intervention message through a social aggregation.

In a separate work (see Kee, Mannucci, Sparks, Struppa, & Damiano, 2011), we discuss seven propositions of how social influence, social aggregations, and leadership groups in social media can be mathematically conceptualized and calculated. We then provide five specific steps for computational modeling of social aggregates in communication networks. In this paper, we calculated social influence and centrality and demonstrate how different higher-dimensional groups within a community can be strategically tapped for accelerating the diffusion of an innovation, such as a health intervention message, for community change. Through this article, we extend the current idea to consider the practical and computational aspects of the simplicial model of social aggregation proposed.

Limitations and Future Research

The simplicial model of social aggregation presented in this article is an attempt to take strongly united social groups into consideration in a communication network.
The simplicial model proposed in its current development has several limitations. First, classical network models typically use directed graphs, also called digraphs or oriented graphs, which allow linkages to have a preferred direction (from A to B or vice versa) and also allow for multiple linkages to exist between the same nodes. Although this concept of orientation can be extended to higher dimensions, we reserve this discussion for future manuscripts and consider only undirected and single linkages between nodes in the present article.

Second, in their discussion of communication network analysis, Contractor et al. (2006) talked about exogenous variables as factors that are “outside” the relations between actors in the network. At the dyadic level, exogenous variables may model different strength levels of friendship between nodes. Although in theory it is possible to extend the notion of simplicial complexes so that its components (nodes, linkages, faces, etc.) also carry “weights” and “attributes,” to focus the scope of this article, the proposed simplicial model does not take into account exogenous variables.

Third, our model only considers higher-dimensional groups of three individuals and more. Conceptually, a dyad can become a higher-dimensional group if it develops uniting conditions beyond simply being connected by a tie (i.e., a casual communication exchange, a business transaction, etc.). Mathematically, when it becomes a higher-dimensional group, it can be represented by a 1-simplex, along with its faces, which are its two 0-simplexes. Therefore, a simple social aggregation (i.e., a simplicial complex) can be created by having a tight social group of two individuals (i.e., a 1-simplex) and a mutual acquaintance (i.e., a 0-simplex). The simplicial model can mathematically distinguish between a group of three pairwise-connected individuals and a social aggregation with a higher-dimensional group of two individuals and a third individual who is a mutual acquaintance. However, visually the simplicial approach cannot adequately distinguish them. Future research should look into addressing this limitation.

Fourth, this article considers the mathematical basis of higher-dimensional groups and social aggregations after relationships among individuals in a community network have already been formed. The article does not take into consideration the question of how new developments in connection-forming behavior might occur, especially given the integration of social media in today's society. Future research should explore how and why such networks form and dissolve over time. These questions are likely to be one of the most important future research efforts related to the present inquiry.

Finally, the proposed approach calls attention to some important ethical considerations. While users voluntarily provide and share personal information on social media, many do so without consciously considering the possibility of their information being mined for purposes other than their own use with others they have allowed into their online networks (Mika, 2005). Therefore, the method proposed in this article could potentially be used by administrators of social media platforms who may surreptitiously compromise the information privacy of their users. Furthermore, when users become aware of such possible privacy violations, they may alter their behaviors and interactions.
Conclusion and Implications

In this article, we built on classical communication network literature to propose a simplicial model of social aggregation that further extends previous work to consider higher-dimensional groups that behave almost like single acting entities in a network. These entities are higher-dimensional because their unity is defined not by strong ties beyond simple one-dimensional node-to-node linkages, but rather by being a tightly bound group (with uniting conditions) mathematically represented by two-dimensional spaces, three-dimensional volumes, four-dimensional geometric spatial elements, etc. In other words, the simplicial model of social aggregation proposed allows for the possibility of modeling three or more individuals as connected, not just two by two but also as a united entity. This simplicial model, and its possibilities, leads us to six implications for communication research.

First, as illustrated in the series of scenarios presented, the simplicial model of social aggregation can be used to model friendship networks on social media. This model can bridge interpersonal communication, nonverbal communication, and computer-mediated communication (CMC) research. For example, Lo (2008) argued that emoticons are communication tools presented in verbal form for nonverbal functions. In another study, Derks, Bos, and von Grumbkow (2008) argued that people use more emoticons in CMC with friends than with strangers. The proposed simplicial model can be used to potentially develop a CMC-based typology of friendship, distinguishing acquaintances, friends, and tightly united friends based on their use of emoticons on Facebook and its various channels.

Furthermore, how social groups reconfigure and evolve over time is also a classic phenomenon in a family context. Family communication researchers have been interested in studying blended families (Braithwaite, Olson, Golish, Soukup, & Turman, 2001), including from a systems perspective (Galvin, Dickson, & Marrow, 2006). The model proposed, when applied with simulation data, can help families and individuals analyze existing family patterns and generate innovative ways to co-create a new future for their blended families as higher-dimensional groups. For example, Baxter, Braithwaite, and Nicholson (1999) studied blended families and identified the most frequent turning points in these social entities to include household configuration, conflict, holidays/special events, and a few other scenarios. By using $C$ to represent a “child,” $M$ a “mom,” $D$ a “dad,” $S$ a “step dad,” $B$ a new “baby” between mom and step dad, and $A$ “another new baby” in early scenarios of college students in this article, we can use the scenarios in the group to represent turning points in a blended family’s household configurations. The conflict between $M$ and $D$ in Scenario 2 can represent a divorce between mom and dad. Each turning point can either move the blended family forward or backward as a higher-dimensional group, under the uniting conditions we theorized. If a simulation of a blended family based on Facebook data reveals that $C$ is not tightly integrated into the blended family represented by a simplex, one can engage in communication intervention strategies, such as tagging all members in a picture taken at a holiday/special event that publicly show $[C, M, S, B, A]$ as a tight entity on Facebook.
Family members can also actively reach out to each other on Facebook via multiple channels, such as private messages, chats, etc., or create a group message on Facebook that everyone can easily “reply to all” to create a private group dialog.

Second, some cultural communities may be more prone than others to form strongly tied social groups. Intercultural communication researchers refer to these societies and communities as having high-context cultures (Gudykunst & Nishida, 1986). High context has also been found to influence communication behaviors and social relationships on the Internet (Würtz, 2005). The simplicial model can be used to study cultural groups, including immigrant groups (Kitai et al., 1983), as they move through the new society, either becoming culturally isolated with members of their home culture (Lum, 1991) or acculturated into the mainstream culture (Kim, 1977) by becoming friends with members of the local culture. Furthermore, in a diverse society, the simplicial model can be used to study urban residential communities (Mattei & Ball-Rokeach, 2003) and racial integration (Matabane, 1988).

Third, group and organizational communication researchers have been interested in studying transactive memory teams (Palazzolo, 2005) in which members use communication (Hollingshead & Brandon, 2003) and technology (Yuan & Gay, 2006) to achieve the state of having a “group mind.” The construct of a transactive memory system is an example of a higher-dimensional group that can be modeled by the simplicial approach. Given the commonplace use of general information technologies (Browning, Sætre, Stephens, & Sørnes, 2008) and the emerging use of social media (McAfee, 2009) in the workplace, this line of research is promising given the extant digital data on organizational networks.

Fourth, in health communication, researchers and practitioners increasingly have become aware of the importance of teamwork among healthcare providers, as communication failures can lead to serious outcomes that put patients’ lives in danger. Medical teams come together as tightly united groups when they socialize and develop (Apker & Eggly, 2004), as well as negotiate and co-manage (Apker, Propp, & Zabava Ford, 2005), their medical professional identity. The simplicial model can be used to model team dynamics to analyze the best combination of healthcare teams when members of a healthcare team change shifts and reassemble at different times.

Fifth, in light of the recent political revolutions sparked by Facebook and Twitter in Tunisia and Egypt, we see the usefulness of the simplicial model in the study of online political deliberation, organizing, and movement (Dahlgren, 2005; Gastil, 2007; Price, Nir, & Capella, 2006). The model we propose is useful for studying the way dispersed strangers come together on a social media platform and become higher-dimensional groups and social aggregation that translate online discussions into offline activities that overturn a political regime.

Sixth, the proposed simulation approach can be used in conjunction with qualitative approaches in a multi-methods study, such as one that employs Howard’s (2002) network ethnography. Howard explains, “Network ethnography is the process of using ethnographic field methods on cases and field sites selected using social network analysis” (p. 561). He further maintains that a researcher can use the results of social network analysis to reveal “subgroups and clusters worthy of close study”
Given the common interests in subgroups and clusters, the simplicial model is highly complementary to network ethnography. The combined results could be computational simulations that also help researchers identify subgroups and clusters embedded in social aggregations for ethnographic studies.

Overall, the simplicial model of social aggregation proposed in this article has multiple theoretical implications for subfields of the communication discipline including, but not limited to, interpersonal, family, nonverbal, group, intercultural, organizational, health, and political communication, as well as qualitative approach of ethnography. Although communication journals were reticent in publishing mathematical models in the early 1980s (McPhee & Poole, 1981), we believe that the timing of this model is right because of the current wealth of digital social data available for such studies. The simplicial model of social aggregation can advance communication research in new ways. This line of research can position the communication discipline as a key contributor to the fields of computational social science (Lazer et al., 2009) and computational and mathematical organization theory (Carley, 1995).

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