Modal Logic for Coalgebras

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Abstract

Coalgebras are of growing importance in theoretical computer science. A major reason is that they model a great variety of dynamic systems. Thus, they provide an excellent opportunity for a unified view on these systems. As a consequence, it is significant to develop languages for coalgebras. Such languages could, for instance, be used to specify and verify systems that are modelled with coalgebras.

Modal logic has proved to be suitable for this purpose. So far, most approaches have presented a language to describe only deterministic coalgebras. The present paper introduces a generalization that also covers non-deterministic systems. We consider coalgebras of so-called Kripke-polynomial functors on the category **Set**. Such functors are inductively constructed from constant functors and the identity functor using product, coproduct, exponentiation by a set, and the power set functor. Kripke-structures turn out to be a special case.

We show how to develop a finitary modal language $\mathcal{L}^F$ for a given Kripke-polynomial functor F. Models of $\mathcal{L}^F$ are the corresponding F-coalgebras. In case of Kripke-structures we obtain the "usual" finitary modal logic.

For so-called image-finite F-coalgebras, the language $\mathcal{L}^F$ is expressive enough to distinguish elements up to bisimilarity, i.e. logical equivalence coincides with bisimilarity. This result even hold for a slightly restricted language. Moreover, we also give a complete calculus for $\mathcal{L}^F$ in case the constants occuring in F are finite.

The key method for this approach is to use a twofold induction where the "outer" one runs on the structure of formulas and the "inner" one on the structure of the functor.