A power algebra of games of choice

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A very simple game of choice is one which starts in a fixed position and whose outcome is determined by a single pair of moves: the player selects a set of possible outcomes, then the adversary selects an element of this set. A game is lost by the participant who is faced with the choice from an empty set: if the family of sets of outcomes from the starting position is empty then the player loses, if the set chosen by the player is empty then the adversary loses. Such games may be viewed as forming a power algebra of the power set Boolean algebra of outcomes.

Allowing a set of possible starting positions yields a more interesting game which may be viewed as a binary multirelation from positions to sets of outcomes. The family of these games has a rich lattice-theoretic structure inherited from the power algebra of games of choice.

With this algebraic view of simple games of choice we invoke (a generalisation of) the techniques of Jónsson/Tarski duality to calculate, given a game and a set of outcomes as winning positions for the player of the game, the initial positions from which the player will win.