Second Order Varieties of Semirings

Abstract

Hyperidentities in our sense were introduced by W. Taylor [Tay; 81] with the aim to extend the concept of identities. Hyperidentities are equations consisting of variables and operation symbols in which one can substitute for the variables, concrete elements of the appropriate algebraic structure and for the operation symbols, concrete term operations of this structure. So they are formulas in a second order language where quantification is allowed as well for individual variables as for operation symbols. For instance one can consider the hyperassociative law as formula of the form:

$$\forall x_1 \forall x_2 \forall x_3 \forall (F(F(x, F(y,z)) \approx F(F(x,y),z))$$

where for the binary operation symbol F any binary term of the considered language can be substituted.

If we request that all identities of a variety are satisfied as hyperidentities, we call this variety solid. Solid varieties are second order varieties. The important fact that all solid varieties of a given type τ form a complete sublattice of the lattice of all varieties of type τ is a helpful tool to study lattices of varieties.

In our paper, we will apply the general theory of hyperidentities to the concrete case of semirings i. e. algebraic structures with two binary associative operations where both distributive laws are valid.

[Tay;81] Taylor, W., Hyperidentities and hypervarieties, Aequationes Mathematicae **23** (1981), 111-127