AN EFFICIENT TRANSFER OF STRONG DUALITIES

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A **full duality** for the quasi-variety $\mathcal{A} := \mathbb{ISP}(\underline{\mathbf{D}})$ generated by a finite algebra $\underline{\mathbf{D}} = \langle D; F \rangle$ is a dual category equivalence between $\mathcal{A}$ and the category $\mathfrak{X} := \mathbb{IS}_c\mathbb{P}^+(\underset{\sim}{\mathbf{D}})$ where $\underset{\sim}{\mathbf{D}} = \langle D; G, H, R, \mathcal{T} \rangle$ is a (discrete) topological structure allowing sets R of relations, G of operations and H of partial operations in its type. Here $\mathbb{I}$, $\mathbb{S}$ $\mathbb{P}$ (arbitrary index sets) and $\mathbb{P}^+$ (non-empty index sets), denote the usual class operators while $\mathbb{S}_c$ denotes "*closed substructure of*". The duality is called **strong** if it is full and moreover $\underset{\sim}{\mathbf{M}}$ is injective in $\mathfrak{X}$. Strong dualities provide a powerful tool for the study of the quasi-variety $\mathcal{A}$. Since a quasi-variety may have many different generating algebras it is useful to know how to take a known strong duality for $\mathcal{A}$ based on a finite generating algebra $\underline{\mathbf{D}}$ and transfer it to a strong duality based on another finite generating algebra $\underline{\mathbf{M}}$. Our concern here is not the *existence* of a strong duality based on $\underline{\mathbf{M}}$: that is known—see Davey and Willard [4] and Saramago [6] for the existence of a duality and Hyndman [5] for the existence of a strong duality. The aim of this paper is to give conditions under which we obtain a particularly efficient and user friendly transfer of a strong duality for $\mathcal{A}$ based on $\underline{\mathbf{D}}$ to a strong duality based on $\underline{\mathbf{M}}$.

We show that, in many cases, if $\underline{\mathbf{D}}$ and $\underline{\mathbf{M}}$ are finite algebras which generate the same quasi-variety $\mathfrak{D}$, then a strong duality for $\mathfrak{D}$ based on $\underline{\mathbf{D}}$ may be transferred to a strong duality for $\mathfrak{D}$ based on $\underline{\mathbf{M}}$ by the addition of some endomorphisms of $\underline{\mathbf{M}}$ and just one further partial operation. This additional operation exhibits the schizophrenia so typical of the theory of natural dualities. We show how the result may be applied to yield an efficient strong duality in the case when $\underline{\mathbf{M}}$ is a distributive lattice, a semilattice or an abelian group.

References

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