On Market Activity and the Value of Money*

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Abstract

In a random-matching monetary economy, efficient and inefficient sellers choose between home or market production. Since inefficient sellers bargain up their prices, two equilibria may exist—with high or low market participation—depending on extent of heterogeneity and frictions. In equilibrium, the presence of inefficient sellers on the market has two opposing effects. It raises trading frequencies, so it lowers consumption risk, but it lowers the value of money, raising prices. This may reduce trading efficiency. Equilibria with full and limited participation can coexist; when average efficiency is high and agents are patient, limited participation is socially preferable.

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1 Introduction

This note contributes to some recent research concerned with the relationship between the equilibrium value of money and the degree of market activity, in heterogeneous economies. This research, developed within the context of models where money has a fundamental allocative role, seems to indicate that—given a money stock—the equilibrium value of money, the endogenous extent of market activity and welfare are all positively correlated.

Johri (1999), for example, modifies the search-theoretic monetary framework of Shi-Trejos-Wright (1995) to show that producers' heterogeneity can sustain equilibrium multiplicity characterized by (full) acceptability of money but different degrees of market activity. The key finding is a positive relationship between the equilibrium level of market activity and the (endogenous) value of money. When money has high value agents consume more and more frequently than when money has less value. In short, a high-valued currency generates positive intensive and extensive margin effects. Shevchenko and Wright's (2004) study of an indivisible-goods heterogeneous agents matching model has similar implications. They endogenize the acceptability of money and find equilibrium multiplicity is possible. In equilibrium, greater acceptability raises the extent of market activity, the value of money and ex-ante welfare (goods are indivisible so prices are unaffected).

A normative implication of these results is that the key to improve the decentralized monetary allocation is to maximize the value of a currency (fostering low prices) because this also implies minimum consumption risk (maximum market activity). Thus, it is natural to ask whether such a positive link between money's value, market activity and welfare, is a general feature of this class of models.

Our study suggests this is not the case. The intuition is that a trade-off may exist between consumption risk and prices. Although expanded market activity has beneficial extensive margin effects—it lowers consumption risk—it may also have negative intensive margin effects—it may lower trading efficiency. For example, prices may substantially rise and trade surplus may fall if greater market activity amounts to adding a small number of very inefficient producers to the sellers' pool.

To provide this intuition we consider a model similar to Johri's, where agents are subject to productivity shocks and choose market trading or home-production, at each date. We study stationary allocations where traded quantities are bilaterally bargained. In equilibrium money's value, prices and market activity are endogenous. Two equilibria may exist, with high or low participation. When agents are patient, money has lots of value so buyers tend to spend it only when they find an efficient producer, who has a low marginal cost. The opposite tends to occur when agents are impatient. Existence also hinges on the available money stock and the distribution of shocks, as they also affect the value of money.

We find equilibrium multiplicity for moderate money supplies and discount factors. Here, money's value (hence traded quantities) is negatively correlated with market activity and may also be negatively correlated with welfare. In fact, we provide examples where ex-ante welfare is *higher* when market activity is the *lowest* and the value of money is *highest*. The reverse, ex-ante welfare is *higher* when market activity is the *highest*, but the value of money is *lowest*, can also occur if agents are more patient. What is the reason?

Low market activity happens when buyers trade only with efficient sellers, who produce a lot for a dollar. Inefficient sellers stay out of the market, which raises consumption risk. These two elements tend to boost the equilibrium value of money. Whether this improves trading efficiency or not depends on just how much prices fall. Since prices fall substantially when agents are more patient and money is more scarce, moderate money stocks and moderate discount factors tend to sustain greater trading efficiency, under low market activity. In addition, if most sellers are efficient, consumption risk grows only slightly, under low market activity. Hence, low market activity may be socially preferred to higher activity. This result tends to be reversed when agents are more patient—since money's value tends to be inefficiently high—and when the seller's pool is mostly inefficient—since greater participation lowers trading risk substantially.

2 Environment

The environment is a version of Shi (1995) and Trejos-Wright (1995). Time is discrete and infinite, $t = 0, 1, 2....$ There is a constant unit mass of people, a divisible and perishable market good, and a home-made good.

At the beginning of each t agents choose market trading or home-production. In the latter case, the agent costlessly produces the home-made good, enjoying $\varepsilon > 0$ consumption utility. In the former case the agent trades on the market as a seller or a buyer, depending on whether he has money or if he can produce market goods. Sellers and buyers are randomly matched. A fraction $M \in (0, 1)$ of agents is endowed with one unit of indivisible fiat money in $t = 0$, while the others can produce market goods once. Everyone can produce market goods contingent on prior market consumption.[1] However, everyone wishes to consume only the market production of someone else; in that case $q > 0$ consumption generates utility $u(q) = \frac{q^\alpha}{\alpha}$, $\alpha \in (0, 1)$. The period discount factor is $\delta \in (0, 1)$.

We introduce heterogeneity by means of random variable costs of production. At the end of each t, an agent draws a productivity shock (i.i.d across time and agents). In the next period he can produce q market goods (i) inefficiently with probability $\lambda \in (0, 1)$, generating disutility θq, or (ii) efficiently with probability $1 - \lambda$, generating disutility $q < \theta q$. This is unlike Johri (1999), who considers random fixed costs of production.

3 Symmetric Stationary Equilibria

We study equilibria where agents adopt identical, time invariant strategies, focusing on pure strategies (it is sufficient to make our point).

At the beginning of each t, an agent can be one of three 'types' k. He is a 'buyer' if he has money, $k = m$, or he is an 'efficient' or 'inefficient seller', $k = H$ or $k = L$, depending on the shock realization. Afterward the agent chooses market trading or home-production. Let p_k be the fraction of traders of type k present on the market in some date t, so that $p_m + p_H + p_L = 1$.

On the market, buyer-seller pairs are randomly formed with the matching rate proportional to the fraction of possible partners. That is, a buyer meets a seller with probability $p_H + p_L$ and a seller meets a buyer with probability p_m. This implies that although some traders may remain unmatched, every match is single-coincidence. Since credit or barter are impossible, agents must exchange money for goods. In a match the seller's type is observed. Let q_H and q_L be the goods traded for money by an efficient and inefficient seller, respectively, in equilibrium. Also, let $c_i(q_i) = \theta_i q_i$ where $\theta_H = 1$ and $\theta_L = \theta$.

3.1 Prices

Define $V_k(q_H, q_L)$ as the beginning-of-period lifetime utility of a type k agent (we omit the arguments when understood). Therefore, for convenience we let

$$F(q_L, q_H) = \delta[V_m - V_H + \lambda(V_H - V_L)]$$

denote the net continuation value of someone who sells today. This is the discounted value of holding money at the beginning of next period, δV_m, minus the discounted expected value of being a seller, $\delta[(1-\lambda)V_H + \lambda V_L]$. Clearly, the buyer's continuation value from buying is $-F(q_L, q_H)$.

In a match with seller $i = H, L$, traders determine the amount Q_i to be exchanged for a unit of money, given $(q_{L,}q_H)$. We assume Q_i is determined according to the Nash bargaining solution, when traders have equal bargaining power and the threat points are their respective lifetime utilities. The match's surplus is $u(Q_i) - c_i(Q_i)$, thus exchange is mutually beneficial if $Q_i \in [0, \hat{Q}_i]$ where $\hat{Q}_i = (\theta_i \alpha)^{\frac{-1}{1-\alpha}}$ solves $u(\hat{Q}_i) = c_i(\hat{Q}_i)$. The seller trades only if his continuation value is larger than his production disutility, $F(q_L, q_H) \geq c_i(Q_i) \geq 0$. Similarly, the buyer trades if $u(Q_i) \geq F(q_L, q_H)$. Since $u(\hat{Q}_i) = c_i(\hat{Q}_i) = \theta_i \hat{Q}_i$, then we must have $\theta_i \hat{Q}_i \geq F(q_L, q_H)$. Symmetric Nash bargaining implies

$$\begin{aligned} q_i = Q_i = \quad & \arg \max_{Q \in [0, \hat{Q}_i]} \left[F(q_L, q_H) - c_i(Q)\right]^{0.5} \left[u(Q) - F(q_L, q_H)\right]^{0.5} \\ \text{s.t.} \quad & u(Q) \geq F(q_L, q_H) \geq c_i(Q). \end{aligned} \tag{1}$$

If an unconstrained solution exists it must be such that $u(Q_i) > F(q_L, q_H) > c_i(Q_i)$. Hence, it must be such that $Q_i \in (0, \hat{Q}_i)$ and $0 < F(q_L, q_H) < \theta_i \hat{Q}_i$. Given (q_L, q_H), the symmetric unconstrained Nash solution is

$$q_i = Q_i \in (0, \hat{Q}_i) \text{ such that } T_i(q_L, q_H, Q_i) = 0 \tag{2}$$

where $T_i(q_L, q_H, Q_i)$ is the first order condition

$$\begin{aligned} T_i(q_L, q_H, Q_i) &= \left[F(q_L, q_H) - c_i(Q_i)\right] u'(Q_i) - \left[u(Q_i) - F(q_L, q_H)\right] c_i'(Q_i) \\ &= F(q_L, q_H)(Q_i^{\alpha-1} + \theta_i) - \tfrac{1+\alpha}{\alpha} \theta_i Q_i^{\alpha}. \end{aligned} \tag{3}$$

Omitting the arguments q_L and q_H, note that $T_i(\hat{Q}_i) < 0 < T_i(0)$, and $\frac{\partial T_i(Q_i)}{\partial Q_i} < 0$. Hence $T_H(Q) > T_L(Q) > T_H(\theta Q)$ for $Q \in (0, \hat{Q}_i)$. Thus, if an equilibrium exists where prices satisfy (2), then it must be that $q_L < q_H < \theta q_L$. This is intuitive. Given any quantity Q, inefficient sellers earn less surplus than efficient sellers. Since sellers' bargaining power is type-independent, it follows that in equilibrium the efficient sellers must offer the best prices, i.e. the largest quantities.

3.2 Strategies and Value Functions

Given $q_L < q_H < \theta q_L$, if buyers prefer to trade with inefficient sellers, they also trade with efficient sellers. Thus, let β'_L be the probability 0 or 1 that a buyer chooses to trade his money for q_L (β_L is the strategy of everyone else). In a symmetric monetary equilibrium individual optimality and aggregate consistency require

$$\beta_L = \beta'_L = \begin{cases} 1 \ \text{ if } u(q_L) \geq F(q_H, q_L) \\ 0 \ \text{ otherwise.} \end{cases} \tag{4}$$

Let π'_k denote the probability 0 or 1 that a type k agent chooses to trade on the market in a period (π_k is the probability chosen by everyone else). The buyer's expected market payoff is

$$p_H \left[u(q_H) - F(q_L, q_H)\right] + p_L \beta_L \left[u(q_L) - F(q_L, q_H)\right]$$

since with probability p_H he trades with an efficient seller, and with probability $p_L \beta_L$ he trades with an inefficient seller. The surplus is $u(q_i) - F(q_L, q_H)$. Individual optimality and aggregate consistency in equilibrium require

$$\pi_m = \pi'_m = \begin{cases} 1 & \text{if } p_H \left[u(q_H) - F(q_L, q_H)\right] + p_L \beta_L \left[u(q_L) - F(q_L, q_H)\right] \geq \varepsilon \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Of course, a monetary equilibrium requires $\pi_m = 1$. Since $q_H > q_L$, then in equilibrium buyers must be willing to trade at least with an efficient seller. This requires $u(q_H) > F(q_L, q_H)$, i.e. $q_H < \hat{Q}_H$ since the seller at most produces $q_H = F(q_L, q_H)$. It also requires ε sufficiently small (more later).

For a seller of type $i = H, L$, individual optimality and aggregate consistency require

$$\pi_H = \pi'_H = \begin{cases} 1 \text{ if } p_m \left[F(q_H, q_L) - q_H\right] \geq \varepsilon \\ 0 \text{ otherwise} \end{cases} \qquad \pi_L = \pi'_L = \begin{cases} 1 \text{ if } p_m \beta_L \left[F(q_H, q_L) - \theta q_L\right] \geq \varepsilon \\ 0 \text{ otherwise} \end{cases} \tag{6}$$

where $p_m \left[F(q_H, q_L) - c_i(q_i)\right]$ is the seller's expected market payoff. In equilibrium $F(q_L, q_H) > c_i(q_i)$ and ε small are necessary or the seller would choose home production.

The implication of (4)-(6) is that in a monetary equilibrium the terms of trade must give *positive* surplus to both partners, since $u(q_i) > F(q_H, q_L)$ and $F(q_H, q_L) > c_i(q_i)$. There are two consequences. First, $u(q_i) > c_i(q_i)$ in every trade, so that $q_i \in (0, \hat{Q}_i)$, i.e. the Nash solution must be unconstrained.[2] Second, since the match's surplus $u(q) - c_i(q)$ is largest for $i = H$, for any q, then if $\pi_m = 1$ the efficient sellers certainly participate in the market, i.e. $\pi_H = 1$. The key is whether inefficient sellers participate. Consequently, there are two types of monetary equilibria: one in which everyone participates in market trades, $\pi_m = \pi_H = \pi_L = \beta_L = 1$, and the other where there is limited participation, as inefficient sellers choose home-production, $\pi_m = \pi_H = 1 > \pi_L = 0$.[3]

Thus, in a monetary equilibrium the value function must satisfy

$$\begin{aligned} V_m = &\quad p_L \beta_L \left[u(q_L) - F(q_L, q_H)\right] + p_H \left[u(q_H) - F(q_L, q_H)\right] + \delta V_m \\ V_H = &\quad p_m \left[-q_H + F(q_L, q_H)\right] + \delta \left[V_H - \lambda(V_H - V_L)\right] \\ V_L = &\quad \pi_L p_m \beta_L [-\theta q_L + F(q_L, q_H)] + (1 - \pi_L)\varepsilon + \delta \left[V_H - \lambda(V_H - V_L)\right]. \end{aligned} \tag{7}$$

Each right hand side basically has two components: the expected payoff from market trade, and the continuation payoff from not trading. This last component is δV_m, for a buyer, and $\delta \left[V_H - \lambda(V_H - V_L)\right]$ for a seller. Inefficient sellers can always guarantee themselves ε payoff, in equilibrium, by avoiding market participation, $\pi_L = 0$. This choice does not appear in the buyer and efficient seller's case, because in a monetary equilibrium $\pi_m = \pi_H = 1$ is necessary.

The monetary equilibrium distribution of market traders must satisfy

$$\begin{aligned} p_m &= \frac{M}{[\pi_L\lambda+(1-\lambda)](1-M)+M} \\ p_H &= \frac{(1-\lambda)(1-M)}{[\pi_L\lambda+(1-\lambda)](1-M)+M} \\ p_L &= \frac{\pi_L\lambda(1-M)}{[\pi_L\lambda+(1-\lambda)](1-M)+M}. \end{aligned} \tag{8}$$

To see why, note that a fraction M of the population has money, $(1-\lambda)(1-M)$ are efficient sellers, and $\lambda(1-M)$ are inefficient sellers. Recall also that inefficient sellers may choose to avoid market trade, setting $\pi_L = 0$, in which case only money traders and efficient sellers are on the market. Thus, we must have $\pi_L\lambda(1-M)$ in (8). This implies it is easier to meet an efficient seller when $\pi_L = 0$, as $p_H|_{\pi_L=0} > p_H|_{\pi_L=1}$.

An equilibrium is defined as follows

Definition. *A monetary equilibrium is a time-invariant list* $\{V_k, p_k, \pi_k, \beta_L\}_{k=m,H,L}$ *that satisfies (4) through (8), where the price in a match is* q^{-1} *and satisfies (1).*

Notice that (4) and (6) imply that if in equilibrium buyers do not trade with inefficient sellers, $\beta_L = 0$, then these sellers prefer to stay out of the market, $\pi_L = 0$. Their behavior affects the decentralized allocation along an *extensive* and an *intensive* margin, because it changes the buyers' matching probabilities and also the distribution of prices.

The extensive effect is negative. Since $p_m|_{\pi_L=0} > p_m|_{\pi_L=1}$, limited participation raises the matching probability of sellers, at the expense of buyers'. Since it is harder to find a seller, buyers experience greater consumption risk.[4] The intensive effect can be positive. If $\pi_L = 0$ a buyer gets to trade only with an efficient seller who offers better prices, $q_H > q_L$. Since these effects move in opposite directions there is potential for equilibrium multiplicity, discussed next.

4 Equilibria

Before proceeding with the analysis, we note that gains from trade are finite, since $q_i < \hat{Q}_i$, and decrease in q_i beyond some $\bar{q}_i \in (0, \hat{Q}_i)$, since $u(q) - c_i(q)$ is hump-shaped. As seen earlier, in a monetary equilibrium the terms of trade must give positive surplus to both partners. From (5) and (6) we see that a monetary equilibrium exists only if ε is small enough. How small? It necessarily must be smaller than the gains from trade *expected* by buyers and (at least) efficient sellers. Since trade opportunities arise stochastically but vanish as $M \to 0, 1$ (see (8)) then M must be bounded away from zero and one, given any $\varepsilon > 0$.[5] Of course, as $\varepsilon \to 0$ then a monetary equilibrium can be sustained for any $M \in (0, 1)$. Thus, to avoid unnecessary complications, in the remainder of the paper we simply focus on the case where $\varepsilon \to 0$. Given this, we have:

Proposition. *Consider $\varepsilon \to 0$. Two types of monetary equilibria can exist:*

(i) a high-participation equilibrium, $\pi_m = \pi_H = \pi_L = 1$, with great frequency of trade $M(1-M)$, and a two-point price distribution associated to the unique pair (q_H^*, q_L^*) with $q_L^* < q_H^*$;

(ii) a low-participation equilibrium, $\pi_m = \pi_H = 1 > \pi_L = 0$, with low frequency of trade $M(1-M)\frac{1-\lambda}{(1-\lambda)(1-M)+M}$, and a degenerate price distribution associated to the unique quantity q_H^{**}.

Specifically $\pi_m = \pi_H = 1$ and

$$\pi_L = \begin{cases} 0 & \text{if } \delta > \delta_0 \text{ and } M \leq M_0(\delta) \\ 1 & \text{if } \delta \leq \delta_1 \text{ or if } \delta > \delta_1 \text{ and } M > M_1(\delta) \end{cases}$$

where $\delta_0 < \delta_1$, $M_1(\delta) < M_0(\delta)$ (these definitions and the proof are in the appendix).

Corollary. *The two monetary equilibria coexist if $\delta \in (\delta_0, \delta_1)$ and $M \leq M_0(\delta)$, or if $\delta \geq \delta_1$ and $M_1(\delta) < M < M_0(\delta)$. In either case $q_L^* < q_H^* < q_H^{**}$.*

There are several important findings. First, two monetary equilibria can exist that are distinguished by the extent of market activity, low or high. If every seller participates in market trade we are in the high-participation equilibrium, otherwise we are in the low-participation equilibrium, where every seller is efficient. Second, each equilibrium type has a unique price distribution. Under high-participation there is price heterogeneity and the efficient sellers offer the lowest prices. The price distribution is degenerate under low-participation. Third, these two equilibria coexist for some parameterization of the model. In this case, the high-participation equilibrium has the least consumption risk, since trade frequencies are high. In short, trading is easier to accomplish than in the low-participation equilibrium.

Now, recall that in this class of models the equilibrium value of money reflects its usefulness in facilitating spot transactions. Thus, the next finding is intuitive. Given coexistence for a certain money stock, money has the *lowest* value when market activity is *high*. It follows that prices are the highest—in every match—when the market is the most vibrant. The reason behind this last result is the existence of a pricing externality. If inefficient sellers enter the market, they do not produce much for a dollar. This lowers the equilibrium value of money. Consequently, efficient sellers *also* raise their prices above those they would otherwise charge. In short, full participation has a negative impact on traded quantities but has a positive impact on trading frequencies.

4.1 Characterization: the Role of Discounting and Money Stock

We characterize existence of monetary equilibrium based on discounting δ and initial money stock M, since these parameters have a direct influence on the value of money.[6] Precisely, money has lots of value when δ is large; in this case sellers are willing to produce a lot in order to get money they can only spend in the future. It is also well known that in this class of models money has great value when it is scarce, M small, and has very little value when M is large. To explain our existence result, we use this intuition, and also a numerical illustration (Figure 1, where $\varepsilon = 10^{-8}$, $\alpha = 0.5, \theta = 3, \lambda = 0.5$).

[FIGURE 1 APPROXIMATELY HERE]

Trace a vertical line through the figure, and start at the bottom of it. When agents are very impatient, sellers do not produce much at all. Hence, if sellers' productivities are not extremely different, a buyer will prefer to spend his money as soon as he gets to meet *any* seller, even if the price is unattractive. Searching for a better price tomorrow is not a good idea because the buyer is impatient to consume. Thus, every match leads to a trade and the market is very active, i.e. $\pi_L = 1$ is the unique equilibrium. This is independent of the money stock.

As we increase the discount factor above $\delta_0(M)$—and the stock of money is not out of hand—the value of money grows enough that buyers prefer to spend their money only in high-value matches, when prices are low. This, however, implies higher consumption risk; if buyers refuse high-price trades then the inefficient sellers prefer to stay out of the market, so that in equilibrium it is harder to buy. Thus, buyers face a trade-off; trade more frequently at higher prices or trade less frequently at lower prices. Since agents make independent and uncoordinated choices—and since buyers cannot spend fractions of their cash holdings—a strategic complementarity exists that generates equilibrium multiplicity, with high or low market activity ($\pi_L = 0, 1$). As discounting grows beyond $\delta_1(M)$, the value of money is so large that spending it to buy just a fistful of goods is never a good idea, i.e. $\pi_L = 0$ is the unique equilibrium.

Why does the money stock matter? Due to random matching, buyers' willingness to pay a high price, rather than waiting to meet a more efficient (and cheaper) seller, depends on the matching frictions, governed by M. Money's value tends to zero as $M \to 1$ because money crowds-out buying opportunities. Thus, the initial money stock cannot be too high, $M < M_1(\delta)$, if $\pi_L = 0$ is an equilibrium. The opposite, $M > M_0(\delta)$, must hold for $\pi_L = 1$ to be an equilibrium, since money has great value when M is small.

5 Market Participation, the Value of Money and Welfare

Since equilibrium multiplicity can arise, a natural question is which equilibrium is socially preferred. Thus, consider ex-ante welfare for $\pi_L = 0, 1$:

$$W(\pi_L) = MV_m(\pi_L) + (1 - M)\left[\lambda V_L(\pi_L) + (1 - \lambda)V_H(\pi_L)\right].$$

The basic result, here, is that welfare in the high-participation equilibrium compares more or less favorably to the low participation equilibrium, depending on the parameters. We provide the relevant intuition with the aid of numerical illustrations, without delving into the less informative mathematical derivations of a formal proof.

To start, pick any initial money stock that sustains equilibrium multiplicity. Now, note that welfare is maximized along two dimensions, trading frequency–or the extensive margin–and trading efficiency–or the intensive margin. We have proved that high-participation generates the highest trading frequency, i.e. it has positive extensive margin effects. We have also proved that money tends to buy a lot in the low participation equilibrium. This affects trade efficiency positively or negatively, depending on how low prices are. Money's indivisibility implies over-production can be as inefficient as under-production. Hence, welfare comparisons hinge on possible trade-offs between intensive and extensive effects of market participation.

High participation is welfare inferior if it slightly improves trading frequencies, but raises prices so much that trades are inefficiently low. The opposite occurs if there are large extensive margin effects; substantially higher trading frequencies may well justify a moderate increase in prices. We provide an illustration via the numerical examples in Figures 2 and 3.

Figure 2 has the parameters of Figure 1 and $\delta = 0.9$. Multiplicities arise for $M \in (0.221, 0.322)$ and the left panel shows ex-ante welfare is uniformly higher under low-participation.

[FIGURE 2 APPROXIMATELY HERE]

The right panel explains why. The trade frequency is not much smaller, compared to high participation. However, average traded quantities are higher and less inefficient than under high participation. The figure reports average surpluses conditional on a match

$$E[S|\pi_L = 1] = \lambda[u(q_L^*) - \theta q_L^*] + (1-\lambda)[u(q_H^*) - q_H^*] \text{ and } E[S|\pi_L = 0] = u(q_H^{**}) - q_H^{**}.$$

Clearly, average surplus is much higher under low participation, so there is a large positive intensive margin effect. Thus, the 'best' equilibrium has the *lowest prices*—the highest value of money—*but also* the lowest market activity.

If we increase δ to 0.99 and λ to 0.95 we get Figure 3.

[FIGURE 3 APPROXIMATELY HERE]

Multiplicities arise for $M \in (0.248, 0.402)$. Since agents are more patient, production under low participation is larger but ex-ante welfare can be smaller for some M (left panel). For those money supplies, trades are most efficient under low-participation, $E[S|\pi_L = 0] > E[S|\pi_L = 1]$, but trading frequencies are extremely low (right panel). The reason is this economy has many inefficient sellers, so their absence from the market causes a severe increase in consumption risk. Thus, high-participation creates large positive extensive margin effects: although trades are smaller, agents get to consume much more frequently. Thus, the best equilibrium has the *highest prices*—the lowest value of money—but the *highest* market activity.

6 Robustness

In this section we briefly discuss alternative price formation mechanisms. In particular, our main concern is whether equilibrium multiplicities would still arise when traders have different degrees of flexibility in making offers. This is an interesting exercise since we have demonstrated that different participation rates affect outcomes via their effect on trading frequencies *and* on the terms of trade. In particular, we have seen that low participation occurs because money is just too valuable so buyers avoid trading with inefficient sellers. For this reason we consider two cases that can be thought of as belonging to the two opposite ends of an imaginary price-flexibility spectrum.

First, we look at an economy without price formation at all; agents simply swap indivisible commodities for indivisible money. Then, we move on to the other extreme when goods *and* monetary offers are fully ex-ante flexible; we do so following Berentsen, Molico and Wright (2002) who allow for contracts with random components in the tradition of Prescott and Townsend (1984).

We start by omitting the possibility of price formation. For instance, suppose sellers H and L produce indivisible goods of different sizes (large or small) or different observable qualities (good and mediocre) so that either way $u(q_L) < u(q_H)$. In this environment the buyer's surplus depends on the fixed quantity q_i and the continuation value $F(q_L, q_H)$. When money is very valuable and q_i is small, the buyer cannot bargain a better price; his only option is to go home hungry, hoping for better luck in the future. Consequently, our earlier intuition applies. We can have $u(q_L) < F(q_L, q_H)$ when $\pi_L = 0$ while $u(q_L) > F(q_L, q_H)$ if $\pi_L = 1$, as low participation may raise $F(q_L, q_H)$. In short, we still expect multiplicity. In fact, since prices cannot adjust at all, mixed trading strategies can also arise if $u(q_i) = F(q_L, q_H)$. Thus, the model with fixed terms of trade should generate an even *greater* richness of monetary equilibria, characterized not only by different participation rates but also by different degrees of acceptability of money.

Now move to the opposite end of the spectrum, when traders bargain not only on output but also on the probability of transferring money. This increases the flexibility in pricing as it convexifies the space of feasible ex-ante price offers *as if* money were divisible.[7] To see why, note that if money is really valuable—and the seller cannot produce much for a dollar—then the buyer can simply pay with small probability. In short, the buyer can lower his *average* expenditure in inefficient trades. Since the probability of spending the dollar can be arbitrarily small, a buyer will never pass onto a consumption opportunity, no matter how valuable is the dollar. In other words, the traders can always find a mutually beneficial agreement (in an ex-ante sense). This simple intuition suggests that with randomized monetary transfers we would still observe price heterogeneity but multiplicities due to market participation should disappear. In particular, participation should always be high in the limit as $\varepsilon \to 0$.[8]

More formally, let $\tau_i \in [0,1]$ denote the probability that the buyer transfers money to seller i in exchange for q_i goods. Here traders bargain over τ_i and $Q \in [0, \hat{Q}_i]$, solving

$$\max \left[\tau_i F(q_L, q_H) - c_i(Q)\right]^{0.5} \left[u(Q) - \tau_i F(q_L, q_H)\right]^{0.5} \text{ s.t. } \tau_i \leq 1.$$

In an unconstrained ($\tau_i \leq 1$) symmetric equilibrium the first order conditions imply:

$$\tau_i = \tfrac{u(q_i)+c_i(q_i)}{2F(q_L,q_H)} \qquad \text{and} \qquad q_i = \left(\tfrac{1}{\theta_i}\right)^{\frac{1}{1-\alpha}}. \tag{9}$$

Of course $\tau_i \leq 1$ only if $F(q_L, q_H) \geq \frac{u(q_i)+c_i(q_i)}{2}$, i.e. money must be sufficiently valuable, otherwise it would be spent with certainty. Observe from (9) that q_i maximizes the match's surplus $u(q_i) - c_i(q_i)$, while τ_i is chosen such that traders share the match's surplus equally, ex-ante:

$$u(q_i) - \tau_i F(q_L, q_H) = \tau_i F(q_L, q_H) - c_i(q_i) = \frac{u(q_i) - c_i(q_i)}{2}.$$

In short, greater flexibility in the buyer's spending strategy makes both traders better off ex-ante, so it never makes sense to avoid a trade with an inefficient seller. The buyer can always limit his 'capital loss' by spending the money infrequently, so that $u(q_i) > \tau_i F(q_L, q_H)$. In addition, welfare will generally be higher than in the case without lotteries. The reason is that by using lotteries traders can maximize the match's surplus and can also increase the incidence of consumption by trading a little something even if the seller is inefficient.

7 Concluding Remarks

We have studied a search-theoretic model of money with heterogeneous sellers and endogenous market participation and shown these features can lead to equilibrium multiplicity, with high or low market activity. Prices and market activity tend to be positively correlated, and the best equilibrium may be the one with low prices but also low economic activity. Money in this case facilitates trades that take place only with the most efficient sellers. However, we suspect equilibrium multiplicities should vanish in models with degenerate distributions of divisible money. In that case, buyers would benefit by spending a little something—instead of their entire money holdings—even in matches with inefficient sellers.

Endnotes

[1] Thus, as in the original search model of money, a buyer always spends his entire cash holdings. This simplifies the distribution of money. Of course, initial money holders must be able to trade with those who do not initially have money. A way to achieve this is to assume the latter agents have a production opportunity and these opportunities can be acquired again only by consuming (as in Shi, 1995, where goods are divisible or Camera, 2000, where goods are indivisible).

[2] A constrained bargaining solution, $u(q_i) \geq F(q_L, q_H) = c_i(q_i)$, violates (6).

[3] If $\pi_L = 0$ we do not need to worry about β_L since out of equilibrium a buyer cannot meet an inefficient seller.

[4] Notice that the number of matches between buyers and sellers, $M(p_L + p_H)$, equals the number of matches between sellers and buyers, $(1-M)p_m(\pi_L\lambda + 1 - \lambda)$, in any equilibrium.

[5] Technically, (8) implies $\lim_{M\to 1}(p_H + p_L) = \lim_{M\to 0} p_m = 0$. Consider an $\varepsilon > 0$ small and a $q_i \in (0, \hat{Q}_i)$. In this case the surpluses from trade and home production are positive. However, the expected gain from market trade is greater than the gain from home production for a buyer and an efficient seller only if $M \in (\underline{M}(\varepsilon), \bar{M}(\varepsilon)) \subset (0,1)$. In this case $\pi_m = \pi_H = 1$ is individually optimal and a monetary equilibrium can be sustained.

[6] We focus on discounting and initial money stock for clarity. In the proof we make it clear existence depends on other parameters, besides δ and M. In particular θ and λ affect the distribution of shocks and the buyers' reservation price. Clearly, if heterogeneity is extreme, say θ is very large, inefficient sellers charge high prices, so buyers would shun them (hence $\pi_L = 0$ always). If heterogeneity is minimal, say, θ close to one or λ close to one, prices are so similar across sellers or there are so few efficient sellers, that buyers would likely buy from the first seller encountered (hence $\pi_L = 1$ always). Technically, δ_1 and δ_0 tend to 1 when $\lambda \to 1$ or $\theta \to 1$.

[7] This is not equivalent to having divisible money as demonstrated in Camera (2005) since agents are constrained to unit holdings. Even greater flexibility can be achieved when agents can hold multiple money units and use randomized trades (see Berentsen, Camera and Waller, 2004). For a model with heterogeneity and lotteries see Lotz, Shevchenko and Waller (2005).

[8] Of course we are modifying the model in two important ways. Since ex-post someone suffers a loss (the seller, if he gets no money and the buyer otherwise), we must assume commitment to a

trade. We also assume market production is independent of market consumption and impose a unit bound on money holdings. The reason is the model in Section 2 does not admit stationary equilibria with randomized money transfers. Since market production is contingent on market consumption, producers who do not receive money are unable to consume (hence produce) market goods and exit the market. As more and more sellers do so p_m reaches an upper bound beyond which money has so little value that it is spent with probability one in every match.

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Appendix

Proof of Proposition. Let $q_i = Q_i \in (0, \hat{Q}_i)$. Using (7)

$$F(q_L, q_H) \quad = \frac{\delta}{A}\left[\frac{p_H q_H^\alpha}{\alpha} + (1-\lambda)\pi_H p_m q_H + \frac{\beta_L p_L q_L^\alpha}{\alpha} + \beta_L \lambda \pi_L p_m \theta q_L - \lambda(1-\pi_L)\varepsilon\right] \tag{10}$$

where $A = 1 - \delta\left[1 - p_H - \beta_L p_L - (1-\lambda)\pi_H p_m - \beta_L \lambda \pi_L p_m\right] > 0$. Notice $\lim_{\varepsilon\to 0} F(q_H, q_L) > 0$. Thus, let $\varepsilon \to 0$.

Conjecture $\pi_m = \pi_H = \pi_L = \beta_L = 1$ is an equilibrium. Note $F(q_L, q_H) < \hat{Q}_H$ and $\frac{\partial F(q_L, q_H)}{\partial q_i} > 0$ for $q_i \in (0, \hat{Q}_i)$. Given q_L, there is a unique $q_H^* \in (0, \hat{Q}_H)$ such that $T_H(q_L, q_H^*) = 0$. The reason is the first and second term in $T_H(q_L, q_H^*)$ cross once. The same argument implies that *if* $F(q_L, q_H) < \theta\hat{Q}_L$, then there is also a unique $q_L^* \in (0, \hat{Q}_L)$ such that $T_L(q_L^*, q_H) = 0$. To verify that the pair (q_L^*, q_H^*) is unique, note it must solve $T_H(q_L, q_H^*) = T_L(q_L^*, q_H) = 0$, which jointly imply

$$\frac{\theta q_L^{*\alpha}}{q_L^{*\alpha-1} + \theta} = \frac{q_H^{*\alpha}}{q_H^{*\alpha-1} + 1}. \tag{11}$$

A unique pair solves this equality and it is such that $q_L^* < q_H^*$ (by continuity, these results hold for $\varepsilon > 0$ small).

We now prove optimality of the conjectured equilibrium strategies. From (6), $\pi_H = 1$ if $\pi_L = 1$. Thus, focus on $\pi_L = 1$, which occurs if $F(q_L, q_H) \geq \theta q_L + \varepsilon/p_M$, which is $F(q_L, q_H) > \theta q_L$for $\varepsilon \to 0$. Since $q_L \in (0, \hat{Q}_L)$ then $u(q_L) > \theta q_L$ so if $u(q_L) \geq F(q_L, q_H)$ then $F(q_L, q_H) > \theta q_L$. That is if $\beta_L = 1$ then $\pi_L = 1$.[1] Thus, focus on proving that $\beta_L = 1$ is individually optimal. If $\pi_H = \pi_L = \beta_L = 1$ then (4) implies $\beta_L = 1$ iff $u(q_L) \geq F(q_L, q_H)$. This inequality is the most stringent for $q_L = \hat{Q}_L$

[1] The intuition is as follows. In a monetary equilibrium a buyer can always buy from an efficient seller. Trade with an inefficient seller occurs only if he can produce enough for a dollar, which can be spent in more efficient trades otherwise. Thus, the buyer's willingness to trade is the determining factor.

(see (10)). Thus, $\beta_L = 1$ iff $u(\hat{Q}_L) = \theta\hat{Q}_L > F(\hat{Q}_L, \tilde{Q}_H)$, where $\tilde{Q}_H$ is the q_H^* that solves (11), given $\hat{Q}_L$. Then, $\theta\hat{Q}_L > F(q_L, q_H)$ iff

$$\delta < \delta_1(M) \quad = \frac{\hat{Q}_L^\alpha}{\lambda\hat{Q}_L^\alpha + (1-\lambda)\left[(1-M)\tilde{Q}_H^\alpha + M\tilde{Q}_H\right]}$$

a function of the parameters. Notice $\frac{\partial\delta_1(M)}{\partial\lambda} > 0$, and $\delta_1(M) = 1$ if $\lambda = 1$. Also $\frac{\partial\delta_1(M)}{\partial M} > 0$, so if $\delta \leq \delta_1(0) = \delta_1$ then $\theta\hat{Q}_L > F(q_L, q_H)$ for all $M > 0$. If $\delta > \delta_1$ then $\theta\hat{Q}_L > F(\hat{Q}_L, \tilde{Q}_H)$ for $M > M_1(\delta) = \frac{\tilde{Q}_H^\alpha - \frac{1-\lambda\delta}{(1-\lambda)\delta}\hat{Q}_L^\alpha}{\tilde{Q}_H^\alpha - \alpha\tilde{Q}_H}$ where $M_1(\delta) \in (0,1)$ for $\delta \in (\delta_1, 1)$. Thus $\beta_L = \pi_L = \pi_H = 1$ if $\delta \leq \delta_1$or $\delta > \delta_1$and $M > M_1(\delta)$.

Conjecture $\pi_m = \pi_H = 1 > \pi_L = \beta_L = 0$ is an equilibrium. Note $F(q_L, q_H) = F(q_H) < \hat{Q}_H$ and $F'(q_H) > 0$ for $q_H \in (0, \hat{Q}_H)$. There is a unique $q_H^{**} \in (0, \hat{Q}_H)$ solving $T_H(q_H^{**}) = 0$,since the first and second term in $T_H(q_H)$ cross once.

To prove optimality of the strategies notice from (6) that $\pi_L = 0$ if $F(q_H) < \theta q_L + \varepsilon/p_M$. For $\varepsilon \to 0$ this implies $\pi_L = 0$ if $F(q_H) \leq \theta q_L$. Since $q_L \in (0, \hat{Q}_L)$ implies $u(q_L) > \theta q_L$, then $u(q_L) \leq F(q_H) \Rightarrow F(q_H) < \theta q_L$. That is if $\beta_L = 0$ then $\pi_L = 0$ for $\varepsilon > 0$ small. Thus focus on proving $\beta_L = 0$. If $\pi_H = 1 > \pi_L = \beta_L = 0$ then (4) implies $\beta_L = 0$ iff $u(q_L) \leq F(q_H)$. This inequality is the most stringent for $q_L = \hat{Q}_L$. Notice that $q_H = \tilde{Q}_H$ solves $T_H(q_H) = 0$ for $u(\hat{Q}_L) = F(q_H)$. Thus $\theta\hat{Q}_L \leq F(\tilde{Q}_H)$ iff

$$\delta \geq \delta_0(M) \quad = \frac{[1-\lambda(1-M)]\hat{Q}_L^\alpha}{M\left[(1-\lambda)\alpha\tilde{Q}_H + \lambda\hat{Q}_L^\alpha\right] + (1-M)(1-\lambda)\tilde{Q}_H^\alpha}.$$

Notice $\frac{\partial\delta_0(M)}{\partial\lambda} > 0$, and $\delta_0(M) = 1$ if $\lambda = 1$. Since $\frac{\partial\delta_1(M)}{\partial M} > 0$, if $\delta < \delta_0(0) = \delta_0$ then $u(\hat{Q}_L) > F(q_H)$ for all $M > 0$. Thus we need $\delta > \delta_0$ for $\pi_L = \beta_L = 0$ and since $\delta_0(1) > 1$ this cannot be satisfied for all M. Rather $\pi_L = \beta_L = 0$ for $\delta > \delta_0$ and $M \leq M_0(\delta) = \frac{\delta\tilde{Q}_H^\alpha - \hat{Q}_L^\alpha}{\delta\left[\tilde{Q}_H^\alpha - \alpha\tilde{Q}_H\right] + \frac{(1-\delta)\lambda}{1-\lambda}\hat{Q}_L^\alpha}$.

It is easily shown that $\delta_0 < \delta_1$, since $\tilde{Q}_H > \hat{Q}_L$, and $M_0(\delta) > M_1(\delta)$ for $\delta < 1$. Thus $\pi_m = \pi_L = \pi_H = 1$ and $\pi_m = \pi_H = 1 > \pi_L = 0$ coexist if $\delta \in (\delta_0, \delta_1]$ and $M \leq M_0(\delta)$ or if $\delta > \delta_1$

and $M_1(\delta) < M < M_0(\delta)$. Notice also that $q_L^* < q_H^* < q_H^{**}$. Since $\theta > 1$ then we have $q_L^* < q_H^{**}$ from (11). Furthermore, one can show $q_H^* < \tilde{Q}_H \leq q_H^{**}$. Also, notice q^{**} is the largest at $M = 0$. In this case $T(q_H^{**}) = 0$ is $u(q_H^{**}) = \frac{1+\alpha-\delta}{\alpha\delta} q_H^{**}$. Since $\frac{1+\alpha-\delta}{\alpha\delta} > 1$ it follows q_H^{**} can be larger than q^*if δ is large and M is small. Using (8) we obtain the frequency of exchange, $(1 - M)p_m[\pi_L\lambda + 1 - \lambda]$.■

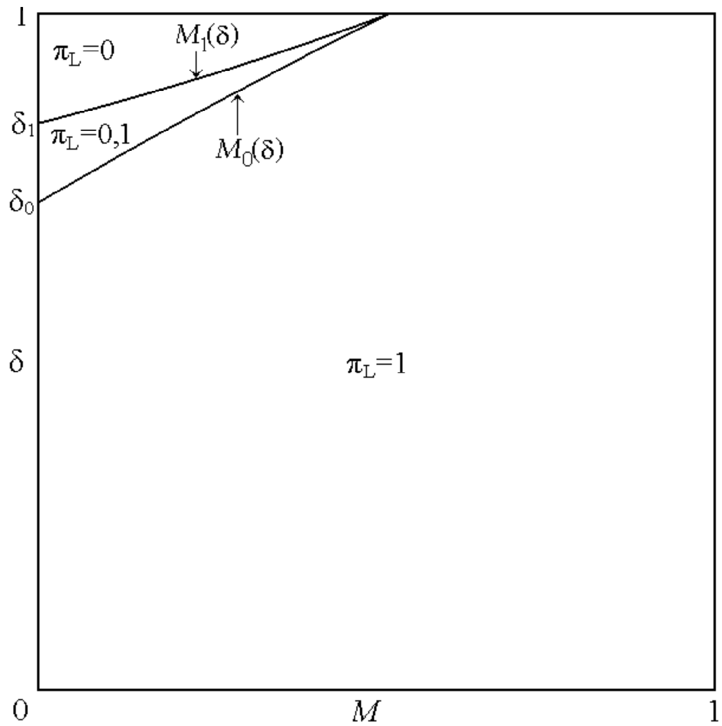

Figure 1: Existence of Equilibria

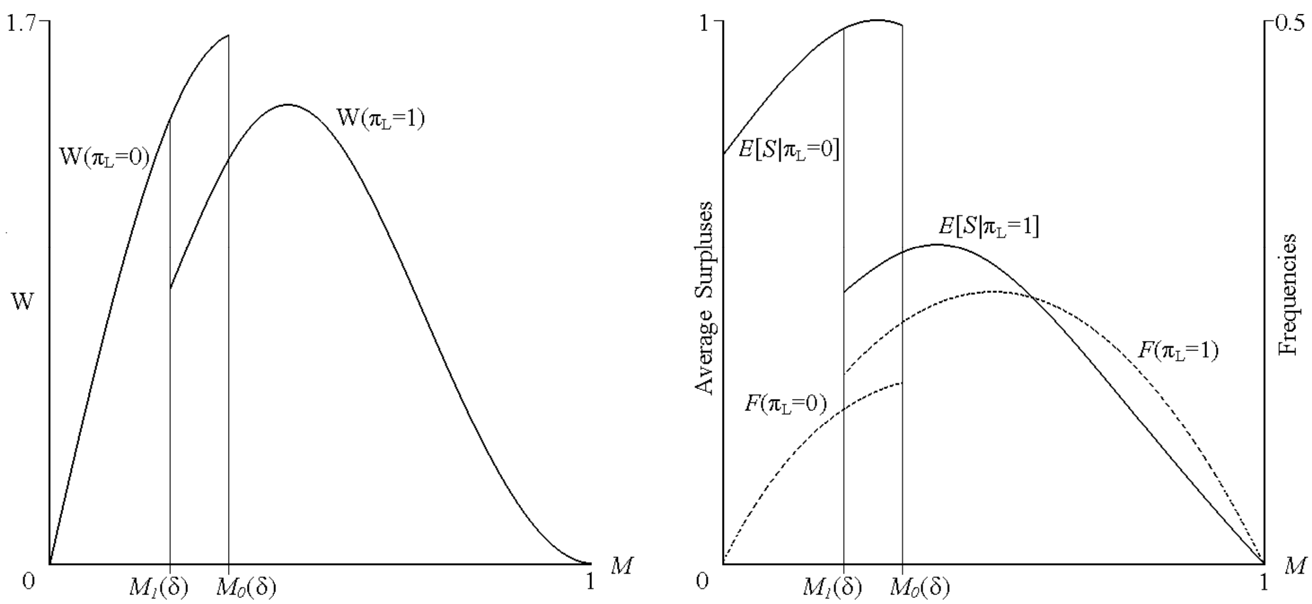

Figure 2: Ex-ante welfare, average surpluses and trading frequencies

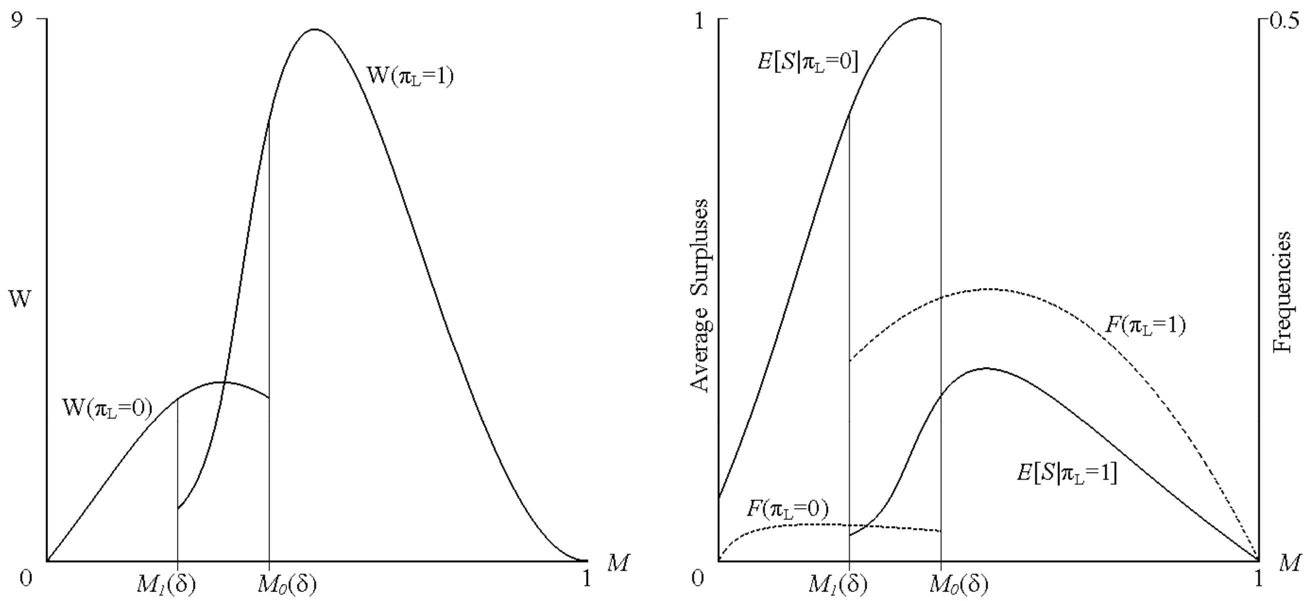

Figure 3: Ex-ante welfare $W(\pi_L)$, average surpluses $E[S|\pi_L]$ and trading frequencies $F(\pi_L)$