Abstract

According to theory, money supports trade in a world without enforcement and, in particular, in large societies, where gift-exchange is unsustainable. It is demonstrated that, in fact, monetary equilibrium breaks down in the absence of adequate enforcement institutions and it collapses as societies that lack external enforcement grow large. This unique result is derived by unveiling the existence of a tacit enforcement assumption in the literature that explains the advantages from monetary exchange, and by integrating monetary theory with the theory of repeated games and social norms.

Keywords: Social norms, repeated games, cooperation, institutions, payment systems.

JEL codes: E4, E5, C7
1 Introduction

Why do societies rely on money? According to theory, the advantage of money is that, by exchanging it, trade can be supported in the absence of enforcement institutions. Put differently, by relying on money individuals are able to outperform equilibria based on rules of voluntary behavior (Hugget and Krasa, 1996; Kocherlakota, 1998; Araujo, 2004).

To develop this idea, imagine a group of anonymous individuals (or strangers) who face repeated opportunities to help others, at a cost. Payoffs are maximized if everyone helps others and minimized if no one helps. If individuals cannot self-commit to actions and external enforcement is unavailable, then they may not trust that help today will be later returned by others, in which case a norm of mutual support may not emerge. Monetary theorists argue that a way to solve this problem is to exchange help for (fiat) money. Yet, the theory leaves open a crucial question. What incentives does monetary exchange provide to help others that norms of voluntary behavior cannot reproduce?

To look into this issue, this paper studies monetary equilibrium by adopting analysis techniques from the literature on repeated matching games (Kandori, 1992; Ellison, 1994). The main findings can be summarized as follows. First, this study unveils the existence of a tacit enforcement assumption in the literature that explains the advantages of monetary exchange. Second, by applying a method of analysis developed in a companion paper (Camera and Gioffré, forthcoming), this study demonstrates that, in fact, monetary equilibrium cannot be sustained without adequate enforcement institutions. In particular, as societies grow large, monetary equilibrium collapses in the absence of some basic form of external enforcement: the enforcement of property rights.

In the model, a stable population of anonymous players is randomly divided in pairs in each period. In every encounter one subject can provide a benefit to the other by
sustaining a small cost (= make a voluntary transfer). This interaction is infinitely repeated (Camera et al., 2013; Camera and Casari, forthcoming). Since players cannot build reputations and cannot adopt relational contracts, there is an incentive to behave opportunistically and avoid making transfers. May the introduction of symbolic objects (= tokens) support an outcome that is socially preferred?

Monetary theorists have offered a positive answer by imposing *quid-pro-quo* constraints: any transfer requires a concurrent payment, or else it fails.¹ In a simultaneous-moves game this amounts to assuming away any temptation to defect (= give nothing) by imposing mechanical punishment (= get nothing), so if money has value, monetary trade is incentive-compatible by design. *Quid-pro-quo* is a form of external enforcement—enforcement of property rights perhaps—which converts the underlying social dilemma into a coordination game by restricting the outcome set.

What if one does not restrict outcomes in a match? The Folk-theorem type results in Kandori (1992) and Ellison (1994) indicate that, in sequential equilibrium, opportunistic behavior must be deterred with proper dynamic incentives or otherwise players might not voluntarily deliver their “quid,” even if they get the “quo.” Now, money no longer sustains exchange without enforcement—external or not—and in particular is no longer capable of providing sufficient incentives to trade if the economy is sufficiently large.

Our finding that money cannot support trade without adequate enforcement is unique. It is meaningful because it provides a theoretical foundation for the notion that monetary exchange cannot operate as a stand-alone institution to overcome trade frictions. Simply put, the option to exchange symbolic objects for goods does not *per se* remove the opportunistic temptations that inhibit cooperation and trade in societies of strangers. In such

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¹For instance, consider the “no-commitment trading mechanism” assumed in Kocherlakota (1998), which embeds a technology that filters out outcomes that are not mutually desirable. For a discussion of *quid-pro-quo* constraints see Starr (1972) or Ostroy and Starr (1990).
societies, individuals do not have high levels of information about others’ past behavior and so basic enforcement institutions must be developed in order for trade to flourish (North, 1991). This leads us to hypothesize that the monitoring difficulties due to the growth in size of human settlements, over the course of history, might have provided a push towards adoption of monetary exchange only in those communities equipped with effective institutions for the enforcement of basic property rights.\(^2\)

The paper proceeds as follows. Section 2 presents the model and reports the main Theorem, which is proved in Section 3. Section 4 offers some final remarks.

2 A model of intertemporal exchange

Consider an economy populated by \( N = 2n \geq 4 \) infinitely-lived agents who face a social dilemma (Camera and Casari, forthcoming; Camera et al., 2013). An exogenous matching process partitions the population into \( n \) pairs in each period \( t = 0, 1, \ldots \). Pairings are random, equally likely, independent over time, and last only one period. Let \( o_i(t) \neq i \) be agent \( i \)'s opponent (or partner) in period \( t \).

In each pair \( \{i, o_i(t)\} \), a coin flip assigns the role of buyer to one agent, and seller to the other. Hence, in each period an agent is equally likely to either be a seller meeting a buyer, or a buyer meeting a seller. The buyer has no action to take. The seller can choose \( C \) or \( D \); \( C \) is interpreted voluntarily transferring a good; Figure 1 reports the payoff matrix, where \( g - d - l > 0 \) and \(-l \leq 0 \leq d < g\).\(^3\)

The outcome \( C \) is called gift-giving: the buyer earns surplus \( g - d \) and the seller’s surplus loss is \(-l \). The outcome \( D \) is called autarky, as it generates no trade surplus.

Define the (socially) efficient outcome in a match as the one in which, giving equal weight

\(^2\)How do such institutions emerge and how do they affect the process of exchange? The work in Kimbrough et al. (2008, 2010) offers some intriguing empirical evidence on this important open question.

\(^3\)E.g., sellers have a perishable good, and buyers derive greater utility than sellers from consuming goods.
\begin{tabular}{c|c|c}
  & \textit{C} & \textit{D} \\
\hline
\textbf{Buyer} & \(g, d-l\) & \(d, d\) \\
\end{tabular}

\textbf{Figure 1:} Interaction in a match

Notes: Row player is a buyer, column player is a seller. Payoffs to (buyer, seller).

to players, total surplus is maximized. Gift-giving is efficient, because \(g - d - l > 0\), but
is \textit{not} mutually beneficial, because buyers benefit at the expense of sellers. Autarky is
the unique Nash equilibrium of a one-shot interaction.

Now consider infinite repetition of such interaction. It is assumed that, in each \(t\), each
agent in \(\{i, o_{i}(t)\}\), for \(i = 1, \ldots, N\), observes only the outcome in their match (= private
monitoring). The identity of \(o_{i}(t)\) and the outcome in other pairs are unobservable, so
players cannot recognize past opponents if they meet them again (= anonymity). These
assumptions imply that agents can neither build a reputation nor engage in relational
contracting—a standard assumption in monetary theory.

Payoffs in the repeated game are the sum of period-payoffs, discounted by a common
factor \(\beta \in [0, 1)\). In the repeated game, the efficient outcome corresponds to the one
in which total surplus is maximized in each match, and in each period. This outcome is
called “gift-giving” because it involves an infinite sequence of unilateral transfers.

Consider a strategy described by a two-state automaton with states “active” and
“idle.” The agent takes actions only as a seller. At the start of any date, if seller \(i\) is
active, he selects \(C\), and otherwise \(D\). Agent \(i\) is active on date \(t = 0\), and in all \(t \geq 1\) (i)
if agent \(i\) is active, then \(i\) becomes idle in \(t + 1\) only if the seller in \(\{i, o_{i}(t)\}\) chooses \(D\).
Otherwise, agent \(i\) remains active; (ii) There is no exit from the idle state. If everyone

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4Equivalently, let the economy be of \textit{indefinite} duration where \(\beta\) is the time-invariant probability that, 
after each period the economy continues for one additional period, while with probability \(1 - \beta\), the
economy ends.
adopts this strategy, then the entire group participates in enforcing defections, and gift-giving is a sequential equilibrium if $N$ is sufficiently small (Araujo, 2004). Otherwise, community enforcement does not represent a sufficient deterrent (Kandori, 1992; Ellison, 1994). So, let us add fiat money to study if its use can solve such enforcement problems.

2.1 The game with money

A random fraction $b = \frac{M}{N} \in (0,1)$ of agents is initially endowed with one indivisible, intrinsically worthless token. It is assumed that token holdings are observable, and cannot exceed one; see Kiyotaki and Wright (1993), Kocherlakota (1998), Araujo (2004) for example. The introduction of tokens expands action sets: in addition to other choices he may have, a player with a token must also decide to either keep the token or to give it to his opponent. The left panel in Figure 2 illustrates the game in a monetary match, defined as a meeting where only the buyer has a token. All other meetings are called non-monetary. Players simultaneously choose actions.

Adding tokens does not eliminate any outcomes possible when $M = 0$: to see this, consider strategies that ignore tokens, which brings us back to Figure 1. Adding tokens expands the strategy set. Consider a strategy that can support monetary exchange. Following Ellison (1994), it is represented using a two-state automaton.

**Definition 1 (Monetary trade strategy).** At the start of any period $t$, agent $i$ can be “active” or “idle:” when active and in a monetary match, $i$ transfers his inventory to $o_i(t)$; otherwise, $i$ makes no transfer. The agent starts active on $t = 0$ and in all $t \geq 1$ the following occurs: (i) If agent $i$ is active, then $i$ becomes idle in $t + 1$ only if $\{i, o_i(t)\}$ is a monetary match where someone makes no transfer—otherwise, agent $i$ remains active; (ii) There is no exit from the idle condition.

The outcome that results when everyone adopts the strategy in Definition 1 is called
### Figure 2: Monetary match without and with external enforcement

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Seller</th>
<th>D</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>give</td>
<td>$g, (d-l)\star$</td>
<td>$d, d\star$</td>
<td>$d\star, d$</td>
</tr>
<tr>
<td>keep</td>
<td>$g\star, d-l$</td>
<td>$d\star, d$</td>
<td>$d\star, d$</td>
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</tbody>
</table>

(No external enforcement)

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<td>$d\star, d$</td>
<td>$d\star, d$</td>
<td>$d\star, d$</td>
</tr>
</tbody>
</table>

(With external enforcement)

Notes: Each panel represents the stage game in a monetary match. There is external enforcement when quid-pro-quo exchange is imposed. Each cell reports payoffs to (buyer, seller); the $\star$ by a player’s payoff denotes holdings of a token at the end of the interaction. Without external enforcement (left panel), token holdings depend only on the buyer’s action, while payoffs depend only on the seller’s action. When external enforcement is imposed (right panel), unilateral transfers are impossible and both payoffs and token holdings jointly depend on the actions of the two players.

**Monetary trade.** Under monetary trade transfers occur only in monetary matches—the seller selects $C$ and the buyer selects $give$. These actions are simultaneous and voluntary. There are no transfers in all other matches because a seller selects $C$ only if the buyer has a token, and a buyer with a token selects $give$ only if the seller has no token.

Monetary trade has two components: a rule of desirable behavior (equilibrium) and a rule of *punishment* to be followed if a departure from desirable behavior is observed (off equilibrium). Players start by making transfers in all monetary matches, but stop forever after observing a deviation. Such switch to a “punishment mode” is absent from monetary models, which impose *quid-pro-quo* constraints: every transfer requires a concurrent payment, and unilateral transfers are impossible. This is not an innocuous assumption. It removes from the outcome set any outcome that is not mutually beneficial, which changes the nature of the game, from a social dilemma to a pure coordination game with Pareto-ranked outcomes (right panel in Figure 2). Ruling out opportunistic

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$^5$The monetary trade strategy in the literature is: after any history, the agent in a monetary match makes an unconditional transfer, and no transfer otherwise. Autarky is the outcome in any match where a departure from this strategy occurs; see (Kocherlakota, 1998; Araujo, 2004; Camera and Casari, forthcoming).
temptations in this manner amounts to introducing a technology for the enforcement of spot trades, for example an institution that can formally enforce private property rights at zero cost. Therefore, one can say that if quid-pro-quo is assumed, then external enforcement is being imposed. The model proposed in this paper lifts this assumption—money does not embody an enforcement technology—hence players must rely on an informal enforcement institution to sustain intertemporal exchange.

2.2 Monetary equilibrium

Conjecture that monetary trade is an equilibrium. Consider an agent with \( j = 0, 1 \) tokens at the start of a period. Define the probability \( m_0 \) that someone without money randomly meets a buyer with a token, and the probability \( 1 - m_1 \) that someone with money randomly meets a seller without a token. Since being a seller or a buyer in a meeting is equally likely, and independent of money holdings, one obtains

\[
\begin{align*}
m_0 & := \frac{1}{2} \times \frac{M}{N - 1}, \\
1 - m_1 & := \frac{1}{2} \times \frac{N - M}{N - 1}.
\end{align*}
\]

Recursive arguments imply that the start-of-period equilibrium payoff \( v_j \) satisfies

\[
\begin{align*}
v_0 & = m_0 (d - l + \beta v_1) + (1 - m_0) (d + \beta v_0), \\
v_1 & = m_1 (d + \beta v_1) + (1 - m_1) (g + \beta v_0).
\end{align*}
\]

(1)

Lemma 1. It holds that \( v_1 > v_0 \) always, and \( v_0 \geq \frac{d}{1 - \beta} \) if

\[
\beta \geq \beta_m := \frac{l}{(1 - m_1)(g - d) + m_1 l} \in (0, 1).
\]

The results are obtained by simple manipulation of the equations in (1). Note that
$\frac{d}{1-\beta}$ is the payoff associated to infinite repetition of the static Nash equilibrium (every seller always chooses $D$), which is always an equilibrium of the repeated game. The condition $\beta \geq \beta_m$ is therefore necessary for existence of monetary equilibrium because it ensures that players earn payoffs above those guaranteed by permanent autarky.

Before reporting the main result, it is demonstrated that $\beta \geq \beta_m$ is also sufficient for existence of monetary equilibrium if external enforcement of spot trades is imposed.\textsuperscript{6} In such case, no agent can sustain (in)voluntary losses, so opportunistic behavior is simply assumed away. Consequently, there is no need for community enforcement and a simple history-independent strategy can be considered to support monetary trade: in a monetary match, players make transfers (money or goods) conditional on receiving a concurrent transfer; otherwise sellers choose $D$, and buyers do not make transfers. Under this strategy the following result can be proved.

**Proposition 1 (Monetary equilibrium with external enforcement).** *Assume external enforcement of spot trades. Monetary equilibrium is supported for all $\beta \in [\beta_m, 1)$ and for any $N$.\*

By virtue of having some external enforcement, monetary trade is incentive compatible whenever players prefer trade to autarky. In particular, the seller’s loss must be small relative to the benefit expected from spending the token in the future, which is guaranteed by $\beta \geq \beta_m$; see also Araujo (2004).

Now, remove external enforcement: the condition $\beta \geq \beta_m$ is no longer sufficient to sustain monetary equilibrium because players can suffer involuntary losses. The following holds:

\textsuperscript{6}This does not mean that monetary trade will necessarily emerge because tokens have no intrinsic value, hence permanent autarky is also an equilibrium. Araujo and Guimaraes (2012) studies how agents might coordinate on adopting monetary trade.
Theorem 1 (Existence of monetary equilibrium without external enforcement).

There exists \((l^*, \beta^*) \in [0, g - d) \times (\beta_m, 1)\) such that for \((l, \beta) \in [l^*, g - d) \times [\beta^*, 1)\) monetary trade is a sequential equilibrium. Monetary trade is not a sequential equilibrium as \(N \to \infty\).

Given the lack of enforcement technologies, monetary exchange must rely on some form of community enforcement of defections. Voluntary transfers are made today only if there are sufficient incentives (i) to avoid community enforcement in the future (the requirement on \(\beta\)) and (ii) to participate in community enforcement if someone deviates (the requirement on \(l\)). Hence, in the absence of enforcement technologies, monetary exchange is self-sustaining only if punishment can spread quickly in the economy. But this is impossible if the group is too large.

The rest of the paper constructs the proof of Theorem 1 by studying the incentives to make voluntary transfers in equilibrium (= cooperate) and to punish off equilibrium (= defect). The analysis begins by studying how punishment spreads off equilibrium, and proceeds by calculating off-equilibrium payoffs.

3 Off-equilibrium punishment and payoffs

Consider the start of an arbitrary period \(t\) in which the economy is or goes off the (monetary) equilibrium path. Following well-established terminology in repeated games, active agents are referred to as cooperators as opposed to defectors, who are idle. Suppose the population is partitioned into \(N - k\) cooperators and \(k = 1, \ldots, N\) defectors. For \(k \geq 2\) the economy is off the equilibrium path. Let \(k = 1\) denote the case in which someone defects for the first time in a monetary match, moving the economy off equilibrium. Let

\[
k \in \kappa := (1, \ldots, N)^T
\]
denote the state of the economy at the start of a generic date and define the $N$–dimensional
column vector $e_k$ with 1 in the $k^{th}$ position and 0 everywhere else ($\mathbf{T} = \text{transpose}$).\(^7\)

It can be shown that, in this case, the probability distribution of defectors $t \geq 1$
periods forward is given by $e_k^\mathbf{T}Q^t$ where

$$Q = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & Q_{22} & Q_{23} & Q_{24} & 0 & \ldots & 0 & 0 \\
0 & 0 & Q_{33} & Q_{34} & Q_{35} & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & Q_{N−1,N−1} \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1
\end{pmatrix}$$

is an $N \times N$ transition matrix with elements $Q_{kk'}$ satisfying $Q_{kk} < 1$ for all $k < N$.

When everyone follows the strategy in Definition 1, the upper-triangular matrix $Q$
describes how contagious punishment spreads from period to period, i.e., how the economy transitions from a state with $k \in \kappa$ to $k' \in \kappa$ defectors. This type of community enforcement has four main properties (Camera and Gioffrè, forthcoming, Theorem 1). First, it is irreversible and contagious: If someone defects today, then tomorrow there cannot be less defectors than today. The number of additional defectors depends on the random matching outcome.\(^8\) Since defection is an absorbing state, the number of defectors expected on any date is greater if the economy starts with more defectors, and can only increase over time. A single defection eventually leads to 100% defectors, an absorbing state that is reached in finite time almost surely.

Suppose there are $k \geq 2$ defectors. Let $v_j(k), j = 0, 1,$ be the payoff to a generic
defector $i$ at the start of $t$ when $k = 2, \ldots, N$. To construct $v_j(k)$, one must compute
earnings/losses that $i$ expects in $t$, and so one must consider all possible encounters in

\(^7\)To be precise, $k = 1$ denotes the state of the economy after matching takes place in equilibrium, when someone defects in a monetary match. This slight abuse in notation is made for convenience. Also note that $y_j \in y := (y_1, \ldots, y_N)$ is used to denote a generic element of vector $y$.
\(^8\) $Q_{12} = 1$ by definition. The first line of $Q$ represents the case when someone defects in a monetary match, in equilibrium.
which $i$ may take part. Indeed, $i$’s opponent in period $t$, $o_i(t)$, may or may not be a cooperator, and may or may not have money. Player $i$’s continuation payoffs in each possible encounter $\{i,o_i(t)\}$ must also be computed. Such payoffs depend on money holdings of $i$, and the number of defectors $k'$ in the continuation game, which depends on the outcome in the match $\{i,o_i(t)\}$ and all other matches. For this reason it is convenient to proceed by considering the probability of each possible encounter $\{i,o_i(t)\}$ that is conditional on reaching a specific $k' \geq k$, where $Q_{kk'}$ is the probability of reaching $k'$ starting from $k \geq 2$.

The analysis proceeds by constructing $v_0(k)$, the payoff to defector $i$ when he has no money at the start of date $t$. Consider any outcome in $t$ leading to $k' \geq k$ defectors in the continuation game. Focus on a match between defector $i$ and $o_i(t)$. Conditional on $k'$ being the state on $t+1$ and $k$ being the state on $t$, let $\alpha_{kk'}^0$ denote the probability that, on date $t$, $o_i(t)$ is a cooperating buyer with money when $i$ has no money (the 0 superscript).\footnote{9}$\alpha_{kk}^0 = 0$ because, if the state does not change ($k' = k$), then it must be the case that no idle seller meets a buyer who cooperates. The probability $\alpha_{kk'}^0$ depends on the distribution of money across defectors. Hence, with probability $\alpha_{kk'}^0$, agent $i$ gains a token and earns $d$; with complementary probability $1 - \alpha_{kk'}^0$, agent $i$ either meets a defecting buyer who has money or is not in a monetary match, and in either of these circumstances $i$ does not receive a token and earns $d$. Using a recursive formulation for $k = 2, \ldots, N$, one obtains

$$v_0(k) = \sum_{k' = k}^{N} Q_{kk'} \{ \alpha_{kk'}^0 [d + \beta v_1(k')] + (1 - \alpha_{kk'}^0) [d + \beta v_0(k')] \}$$

$$= d + \beta \sum_{k' = k}^{N} Q_{kk'} v_0(k') + \beta \sum_{k' = k}^{N} Q_{kk'} \alpha_{kk'}^0 [v_1(k') - v_0(k')] .$$

(2)

It holds that $v_0(N) = \frac{d}{1 - \beta}$ because $\alpha_{kk'}^0 = 0$ for all $k'$ (it is impossible to meet a cooperator when everyone is a defector). In addition, since $v_1(k) \geq v_0(k)$ for all $k$ (which is shown later), $v_0(k) \geq v_0(N) = v_a$ also holds for all $k = 2, \ldots, N$.\footnote{12}
The payoff to someone who has no money and defects in equilibrium is

\[ v_0(1) = d + \beta v_1(2), \]

which follows from the fact that \( Q_{12} = 1 \) and \( \alpha^0_{12} = 1 \) by definition (if a seller is the first player to defect, \( k = 1 \), then he must defect in a monetary match, in which case he surely meets a cooperating buyer and the economy transitions to \( k' = 2 \)).

Now, \( v_1(k) \) is constructed, which is the payoff to defector \( i \) when he has money at the start of \( t \). Consider any outcome in \( t \) leading to \( k' \geq k \) defectors in the continuation game starting on \( t + 1 \). Conditional on reaching \( k' \) and \( k \) being the current state, let \( \alpha^1_{kk'} \) denote the probability that \( o_i(t) \) is a cooperating seller without money.\(^\text{10}\) Hence, with probability \( \alpha^1_{kk'} \), agent \( i \) keeps his token and earns \( g \); with complementary probability \( 1 - \alpha^1_{kk'} \), defector \( i \) either meets a defecting seller who has no money, or is not in a monetary match, and in either case, \( i \) keeps the token and earns \( d \). Consequently, for \( k = 2, \ldots, N \) one obtains

\[
\begin{align*}
v_1(k) &= \sum_{k' = k}^{N} Q_{kk'} \{ \alpha^1_{kk'} [ g + \beta v_1(k') ] + (1 - \alpha^1_{kk'}) [ d + \beta v_1(k') ] \} \\
&= \sigma_k g + (1 - \sigma_k) d + \beta \sum_{k' = k}^{N} Q_{kk'} v_1(k').
\end{align*}
\]

The second line above follows from observing that—when there are \( k \) defectors in the economy—the unconditional probability that \( i \) (who has a token) is in a monetary match with a cooperating seller is

\[
\sigma_k := \sum_{k' = k}^{N} Q_{kk'} \alpha^1_{kk'} = \sum_{k' = k+1}^{N} Q_{kk'} \alpha^1_{kk'} \quad \text{for } k = 2, \ldots, N.
\]

If \( h \geq k \), then \( \sigma_k \geq \sigma_h \) (with more defectors, meeting cooperators is less likely).

\(^\text{10}\)Here \( \alpha^1_{kk} = 0 \) because, by definition, no defector meets a cooperator in this case.
It is convenient to define $\sigma_1 = 1$ and the vector

$$\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{N-1}, 0)^T.$$ 

For $k \geq 2$, each element $\sigma_k$ defines the probability that buyer $i$ is in a monetary match and meets a cooperator, given that $i$ is one of $k$ defectors at the start of a period. It should be clear that $0 = \sigma_N < \sigma_{k'} < \sigma_k < \sigma_1 = 1$ for $2 \leq k < k' \leq N - 1$. Also, define

$$v_1(1) = g + \beta v_1(2),$$

as the payoff to a buyer who is in monetary match \textit{in equilibrium}, and defects. Define

$$\phi_k = (1 - \beta)e_k^T(I - \beta Q)^{-1}\sigma$$

for $k = 1, \ldots, N$.

Following Camera and Gioffré (forthcoming), it can be interpreted as the expected number of cooperators without money that a defector with money meets in the continuation game, normalized by $(1 - \beta)^{-1}$.

**Lemma 2.** It holds that

$$v_1(k) = \frac{1}{1 - \beta}[\phi_k g + (1 - \phi_k)d] \quad \text{for } k = 1, \ldots, N,$$

(5) with $v_1(k)$ non-increasing in $k$ and $\lim_{\beta \to 1^-} \frac{\phi_k}{1 - \beta} < \infty$.

The proof immediately follows from Camera and Gioffré (forthcoming, Theorem 2). Having characterized payoffs in and out of equilibrium, the analysis proceeds by studying deviations in and out of equilibrium.
3.1 Equilibrium deviations

Suppose everyone has been active until period $t$ and in period $t + 1$ agent $i$ deviates, reverting to play the monetary trade strategy on $t + 2$. Agent $i$ meets cooperator $o_i(t)$ who may or may not have money.

In a non-monetary match, player $i$ does not deviate by making a transfer. Doing so is suboptimal because $i$ has a loss but no future gain (continuation payoffs do not change because his opponent remains active). Hence, consider a monetary match in equilibrium. Two cases may occur. In the first case, player $i$ has no money and $o_i(t)$ is a buyer with money. If $i$ deviates by choosing $D$, then $i$ receives money and his opponent becomes idle. Such deviation is suboptimal if

$$d + \beta v_1(2) \leq d - l + \beta v_1.$$  \hspace{1cm} (6)

In the second case, player $i$ has money and $o_i(t)$ is a seller without money. If $i$ keeps his token, then he obtains $g$ and $o_i(t)$ becomes idle. Such deviation is suboptimal if

$$g + \beta v_1(2) \leq g + \beta v_0.$$  \hspace{1cm} (7)

**Lemma 3 (No deviations in equilibrium).** There exists a value $\beta^* < 1$ such that (6)-(7) hold for all $\beta \in [\beta^*, 1)$.

**Proof of Lemma 3.** In Appendix.

The proof of the Lemma reveals that the key participation constraint is the buyer’s. In equilibrium, if the buyer pays a seller — which occurs if $\beta$ is sufficiently large — then it is also true that the seller serves the buyer. The reverse, however, is not true.

**Lemma 4 (Large economies).** Monetary trade is not an equilibrium as $N \to \infty$.

**Proof of Lemma 4.** In Appendix.
Intuitively, in large economies buyers prefer to avoid paying because they can immediately consume, cannot be immediately punished, \textit{and} can spend their money in the future. This destroys the value of money. The conclusion is that monetary equilibrium cannot generally be sustained when monitoring and punishment limitations preclude adequate enforcement—external or not. This is true for the same reason it is true for social norms: in equilibrium individuals \textit{voluntarily} sustain a loss to provide a benefit to others only if group punishment is a significant threat. The next section studies the credibility of the threat, by considering the incentives to punish off-equilibrium.

\subsection{Off-equilibrium deviations}

Community enforcement is credible if actions in the punishment phase are individually optimal. Deviating in non-monetary matches is suboptimal (the deviator has a loss and cannot slow down contagion). However, a defector might wish to deviate in a monetary match, to slow down the contagious spread of punishment. This case is studied, next.

Suppose there are $k \geq 2$ defectors, and agent $i$ is one of them. Let $\hat{v}_j(k)$ denote $i$’s payoff when he has $j = 0, 1$ tokens. Deviating is suboptimal if

$$\hat{v}_j(k) \leq v_j(k).$$

To derive $\hat{v}_j(k)$, consider that the expected payoffs from not punishing depend on the probabilities of meeting a defector with and without money.

Consider defector $i$ when he does not have money. Conditional on $k'$ being the state next period and $k$ currently, let $\mu^0_{kk'}$ denote the probability that $o_i(t)$ is a buyer with money and $\delta^0_{kk'}$ the conditional probability that $o_i(t)$ is a defecting buyer with money, so

$$\alpha^0_{kk'} + \delta^0_{kk'} = \mu^0_{kk'}, \quad \text{and} \quad \sum_{k' = k}^N Q_{kk'} \mu^0_{kk'} = m_0 \quad \text{for all } k \geq 2.$$
Similarly, consider defector $i$ when he has money. Let $\mu_{kk'}^1$ denote the conditional probability that $o_i(t)$ is a seller without money, and $\delta_{kk'}^1$ the probability that $o_i(t)$ is a defecting seller without money. Hence,

$$\alpha_{kk'}^1 + \delta_{kk'}^1 = \mu_{kk'}^1,$$

and

$$\sum_{k' = k}^{N} Q_{kk'} \mu_{kk'}^1 = 1 - m_1 \quad \text{for all } k \geq 2.$$

Using a recursive formulation:

$$\hat{v}_0(k) = \sum_{k' = k}^{N} Q_{kk'} \alpha_{kk'}^0[d - l + \beta v_1(k' - 1)] + \sum_{k' = k}^{N} Q_{kk'} \delta_{kk'}^0[d - l + \beta v_0(k')]$$

$$+ \sum_{k' = k}^{N} Q_{kk'} (1 - \mu_{kk'}^0)[d + \beta v_0(k')]$$

$$\hat{v}_1(k) = \sum_{k' = k}^{N} Q_{kk'} \alpha_{kk'}^1[g + \beta v_0(k' - 1)] + \sum_{k' = k}^{N} Q_{kk'} \delta_{kk'}^1[d + \beta v_0(k')]$$

$$+ \sum_{k' = k}^{N} Q_{kk'} (1 - \mu_{kk'}^1)[d + \beta v_1(k')]$$

To derive these expressions note that $i$ deviates only in a monetary match. If $i$ has no money, then consider $\hat{v}_0(k)$. The first line accounts for the cases in which $i$ is in a monetary match (i.e., a seller). Here, the agent earns $d - l$ because he cooperates instead of punishing. Player $i$ might also receive money, but this depends on whether his opponent (who is a buyer with money) cooperates or defects. If his opponent is a cooperator, then $i$’s continuation payoff is $v_1(k' - 1)$, because this cooperator does not become a defector ($i$ cooperates) and gives money to $i$. Otherwise, the continuation payoff is $v_0(k')$ because there is no impact on the number of future defectors and $i$ does not receive money.

The second line defines matches in which $i$ is not in a monetary match. The probability of not meeting a buyer with money can be decomposed as

$$1 - m_0 = \sum_{k' = k}^{N} Q_{kk'} (1 - \mu_{kk'}^0),$$

because, conditional on transitioning from $k$ to $k'$, the probability of not meeting a buyer with money is $1 - \mu_{kk'}^0$. In all these matches player $i$ earns $d$ and does not receive money.
so the continuation payoff is \( v_0(k') \), depending on the realization of \( k' \).

If \( i \) has money, instead, then consider \( \hat{v}_1(k) \). Here agent \( i \) deviates only when he is in a monetary match (i.e., a buyer), which is reported in the first line. Deviating means that \( i \) transfers money instead of keeping it. If his opponent is a cooperator, then \( i \) earns \( g \) and the continuation payoff is \( v_0(k' - 1) \) because his opponent does not observe a defection; otherwise \( i \) earns \( d \) and gets \( v_0(k') \) continuation payoff. The second line refers to the matches that are not monetary.

Now, use the definitions of \( v_j(k) \), for \( k = 2, \ldots, N \), to derive inequalities that guarantee that single-period deviations are not profitable, out of equilibrium. Deviating from punishment in a monetary match is suboptimal for defector \( i \), when \( i \) is a buyer, if \( \hat{v}_1(k) \leq v_1(k) \), which is rewritten as

\[
\sum_{k'=k}^{N} Q_{kk'} \alpha_{kk'}^1 [v_0(k' - 1) - v_1(k')] \leq \sum_{k'=k}^{N} Q_{kk'} \delta_{kk'}^1 [v_1(k') - v_0(k')],
\]

(8)

by manipulating expressions \( v_1(k) \) and \( \hat{v}_1(k) \) and using the fact that \( \alpha_{kk'}^1 + \delta_{kk'}^1 = \mu_{kk'}^1 \).

**Lemma 5 (Buyers punish).** Inequality (8) holds for all \( \beta \in (0, 1) \).

**Proof of Lemma 5.** In Appendix. \( \Box \)

Out of equilibrium, it is never optimal for a buyer to deviate from the punishment strategy. The reason is simple. Suppose buyer \( i \) today pays the seller, when in fact he should not. Paying the seller may slow down the growth in the number of defectors but agent \( i \) cannot benefit from it until he re-acquires money. Hence, since future payoffs are discounted, it is a dominant strategy to not pay out of equilibrium.

Deviating from punishment in a monetary match is suboptimal for defector \( i \), when \( i \)
is a seller, if \( v_0(k) \leq v_0(\tilde{k}) \). Using \( \alpha^0_{kk'} + \delta^0_{kk'} = \mu^0_{kk'} \), the inequality is

\[
\beta \sum_{k'=k}^{N} Q_{kk'} \alpha^0_{kk'} [v_1(k' - 1) - v_1(k')] \leq l \sum_{k'=k}^{N} Q_{kk'} \mu^0_{kk'}.
\]  

(9)

Lemma 6 (Sellers punish). There exists \( 0 \leq l^* < g - d \) such that if \( l \in [l^*, g - d) \), then inequality (9) holds for all \( \beta \in (0, 1) \).

Proof of Lemma 6. In Appendix

Out of equilibrium, cooperating as a seller may slow down the growth in defectors. This benefits the seller because he acquires money and may be able to spend it tomorrow. To remove the incentive to deviate, the seller’s loss from making a unilateral transfer must be sufficiently high.

4 Additional results

The previous analysis immediately implies that if monetary equilibrium and gift-giving equilibrium coexist, then gift-giving generates the greatest welfare since it creates surplus in every match, unlike monetary equilibrium. It is therefore natural to ask whether money supports trade on a larger segment of parameters compared to gift-giving. Perhaps this is the advantage of monetary trade. Unfortunately, this is not necessarily so because there can be greater incentives to defect in monetary equilibrium than in gift-giving equilibrium, as is shown next.

Proposition 2. When gift-giving is an equilibrium and players are sufficiently impatient, monetary equilibrium may not exist for some values of the money supply.

Proof of Proposition 2. In Appendix
To provide intuition, note that the money supply determines the frequency of monetary matches, where defections can take place. Consider the extreme case in which there is only one token in the economy, hence there can be at most one monetary match in a period (there could be zero). If the buyer moves off monetary equilibrium, then he can “cheat” everyone in the economy until everyone is a defector. This is so because in this one-token economy defecting sellers cannot “infect” cooperating sellers, which is unlike what happens off-equilibrium under gift-giving, where the number of defectors therefore grows much faster. This, and the fact that payoffs in monetary equilibrium are below gift-giving payoffs, implies that the buyer has a greater incentive to deviate compared to gift-giving equilibrium. Hence, for sufficiently low discount factors one can support gift-giving but not monetary trade in equilibrium.

Now notice that if a technology for the external enforcement of spot trades could be adopted, then the use of money would present a clear advantage over gift-giving. In particular, it would enable intertemporal trade in large economies (Proposition 1), where gift-giving would not. How much would society be ready to pay to adopt external enforcement?\textsuperscript{11} To answer this question, consider the limiting case of a large economy and suppose that, before tokens are distributed and shocks realized, individuals can pay a per-capita cost \( c \geq 0 \) to create a permanent capability to enforce spot trades.\textsuperscript{12} The following result holds.

**Proposition 3.** Consider a large economy with a per-capita supply of tokens \( b \in (0, 1) \) and a per-capita cost \( c \geq 0 \) of adopting a technology for enforcing spot trades. Monetary

\textsuperscript{11} We thank an anonymous referee for raising this point.

\textsuperscript{12} There are related studies about cooperation and optimal risk-sharing arrangements in prolonged economic relationships where agents can exploit the use of costly enforcement technologies; for example, see Krasa and Villamil (2000), Dixit (2003), and Koeppl (2007).
equilibrium exists for all \((c, \beta) \in \mathbb{R}_+ \times [\beta_m, 1)\) that satisfy

\[
c(1 - \beta) \leq \frac{(1 - b)b}{2} \times (g - d - l).
\]

**Proof of Proposition 3.** In Appendix

The main message is that, if the cost \(c\) is sufficiently small, then we are back in Proposition 1. Otherwise, monetary equilibrium exists on a subset of the discount factors considered in the literature. Intuitively, in choosing whether to adopt an enforcement technology, the agent compares the (normalized) cost \(c(1 - \beta)\) to the present discounted value of the benefits expected from having external enforcement. The benefits are displayed on the right hand side of the inequality in Proposition 3, where \(\frac{(1-b)b}{2}\) is the frequency of monetary matches and \(g - d - l\) is the surplus in a monetary match. It is immediate that monetary equilibrium exists for all \(\beta \geq \beta_m\) if \(c\) is sufficiently small. Otherwise, \(\beta > \beta_m\) is needed because as the cost of external enforcement increases, agents must be compensated with greater continuation payoffs from engaging in monetary trade.

5 Final remarks

According to monetary theory, the advantage of monetary exchange lies in its unique ability to support trade in a world where norms of voluntary behavior alone cannot sustain it. This notion hinges on a tacit assumption: money embodies a technology to enforce spot trades, which prevents traders from suffering losses. The present study has demonstrated that without such basic enforcement of property rights hard-wired into the model, monetary exchange is afflicted by the same problems that undermine norms of voluntary behavior. Hence, monetary equilibrium collapses as economies that lack
external enforcement grow large.

The use of money presents a definite theoretical advantage over gift-giving when a technology for the external enforcement of spot trades is available for adoption. If the adoption cost is sufficiently small, then the institution of money supports trade in large economies, whereas gift-giving does not. Recent empirical evidence suggests that there also exist behavioral reasons that make quid-pro-quo monetary exchange attractive compared to gift-giving. Camera et al. (2013) and Camera and Casari (forthcoming) study experimental economies in which (fiat) money is inessential because rules of voluntary behavior are theoretically capable to sustain efficient intertemporal exchange, whereas monetary exchange is not. Yet, they report that monetary exchange endogenously emerges and empirically outperforms non-monetary outcomes as the economies grow large.
Appendix

**Proof of Lemma 3.** Start by considering deviations by a buyer in a monetary match, i.e., (7), which is satisfied if \( v_0 - v_1(2) \geq 0 \). Using the definition of \( v_1(k) \) in (5) one obtains

\[
v_1(2) = \frac{d}{1 - \beta} + \frac{\phi_2(g - d)}{1 - \beta}.
\]

Hence,

\[
v_0 - v_1(2) = \frac{(1 - \beta)v_0 - d}{1 - \beta} - \frac{\phi_2(g - d)}{1 - \beta}.
\]

Using the definition of \( v_0 \) in (1) one obtains

\[
(1 - \beta)v_0 - d \equiv \frac{\beta m_0(1 - m_1)(g - d) - m_0 l(1 - m_1)}{1 + \beta (m_0 - m_1)} > 0 \quad \text{if} \quad \beta > \beta_m,
\]

\[
\lim_{\beta \to 1^-} \frac{(1 - \beta)v_0 - d}{1 - \beta} = \infty.
\]

By Camera and Gioffré (forthcoming, Theorem 3) \( \lim_{\beta \to 1^-} \frac{\phi_2}{1 - \beta} < \infty \). Hence, by continuity \( \beta^* \in (\beta_m, 1) \) exists such that \( v_0 - v_1(2) \geq 0 \) for \( \beta \in [\beta^*, 1) \).

Now, consider deviations by a seller in a monetary match. Inequality (6) is satisfied if \( \beta[v_1 - v_1(2)] \geq l \). From the definition of \( v_0 \) and \( v_1 \) in (1) one obtains

\[
\beta(v_1 - v_0) \equiv \frac{\beta(1 - m_1)(g - d) + m_0 l}{1 + \beta (m_0 - m_1)} \geq \beta_m \frac{(1 - m_1)(g - d) + m_0 l}{1 + \beta_m (m_0 - m_1)} \equiv l \quad \text{for} \quad \beta \geq \beta_m.
\]

Now fix \( \beta \in [\beta^*, 1) \) so that \( v_0 \geq v_1(2) \). Consequently \( v_1 - v_1(2) \geq v_1 - v_0 \geq l/\beta \), i.e., inequality (6) is also satisfied when \( \beta \in [\beta^*, 1) \).\]
Proof of Lemma 4. It is shown that buyers do not wish to pay in economies that are “large.” To define a large economy, let $M = bN$ for $b \in (0, 1)$ and let $N \to \infty$. That is, fix a per-capita money supply and let the economy grow large.

Consider a defector who is a buyer in a monetary match, out of equilibrium when there are $k \geq 2$ defectors. Since the number of defectors is finite, the unconditional probability that the buyer is in a monetary match with a seller who is a cooperator is

$$
\lim_{N \to \infty} \sigma_k = \frac{1}{2}(1 - b).
$$

Hence,

$$
\lim_{N \to \infty} \phi_2 = \lim_{N \to \infty} (1 - \beta)e_2^T(I - \beta Q)^{-1}\sigma = (1 - \beta) \sum_{j=2}^{\infty} (I - \beta Q)^{-1}_{2j} \lim_{N \to \infty} \sigma_j
$$

$$
= \frac{1}{2}(1 - b)(1 - \beta) \sum_{j=2}^{\infty} (I - \beta Q)^{-1}_{2j} = \frac{1}{2}(1 - b),
$$

where $(I - \beta Q)^{-1}_{2j}$ denotes element in row 2 column $j$ of matrix $(I - \beta Q)^{-1}$; $(I - \beta Q)^{-1}_{21} = 0$ because $Q$ is upper triangular.

Recall that a buyer does not deviate in equilibrium if (7) holds, i.e., if $v_0 - v_1(2) \geq 0$. But this is violated for all $\beta \in (0, 1)$ as $N \to \infty$. To see this note that

$$
\lim_{N \to \infty} [v_0 - v_1(2)] = \lim_{N \to \infty} \left[ \frac{(1 - \beta)v_0 - d}{1 - \beta} - \frac{\phi_2(g - d)}{1 - \beta} \right]
$$

$$
= - \frac{[2 - \beta(1 + b)][(g - d)(1 - b) + bl]}{2(1 - \beta)(2 - \beta)} < 0.
$$

Proof of Lemma 5. Prove by contradiction that $v_1(k) \geq v_0(k)$, for all $k = 2, \ldots, N$.

Suppose $v_1(h) < v_0(h)$ for some $2 \leq h \leq N$. Use the definition of $v_0(k)$ in (2) and
notice that $v_j(k) \geq v_j(k + 1)$ for $j = 0, 1$ and all $k = 2, \ldots, N - 1$. It holds that

$$v_0(h) = d + \beta \sum_{k' = h}^N Q_{hk'} \alpha^0_{hk'} v_1(k') + \beta \sum_{k' = h}^N Q_{hk'} (1 - \alpha^0_{hk'}) v_0(k')$$

$$< d + \beta v_1(h) \sum_{k' = h}^N Q_{hk'} \alpha^0_{hk'} + \beta v_0(h) \sum_{k' = h}^N Q_{hk'} (1 - \alpha^0_{hk'})$$

$$< d + \beta v_0(h) \sum_{k' = h}^N Q_{hk'} \alpha^0_{hk'} + \beta v_0(h) \sum_{k' = h}^N Q_{hk'} (1 - \alpha^0_{hk'})$$

$$= d + \beta v_0(h),$$

which provides the desired contradiction because $v_0(k) \geq \frac{d}{1 - \beta}$ for all $k = 2, \ldots, N$.

Consider inequality (8). Prove that it holds whenever $v_1(k) \geq v_0(k - 1)$, for all $k = 2, \ldots, N$. Using the definition of $v_0(k)$ in (2) one obtains

$$v_0(k) = d + \beta \sum_{k' = k}^N Q_{kk'} v_0(k') + \beta \sum_{k' = k}^N Q_{kk'} \alpha^0_{kk'} (v_1(k') - v_0(k'))$$

$$= d + \beta \sum_{k' = k}^N Q_{kk'} v_0(k') + \beta \sum_{k' = k+1}^N Q_{kk'} \alpha^0_{kk'} (v_1(k') - v_0(k'))$$

$$\leq d + \beta \sum_{k' = k}^N Q_{kk'} v_0(k') + \beta \sum_{k' = k+1}^N Q_{kk'} (v_1(k') - v_0(k'))$$

$$= d + \beta Q_{kk} v_0(k) + \beta \sum_{k' = k+1}^N Q_{kk} v_1(k')$$

$$\leq d + \beta Q_{kk} v_0(k) + \beta (1 - Q_{kk}) v_1(k + 1).$$

The fact that $\alpha^0_{kk'} = 0$ when $k' = k$ has been used to derive the second line. For the third line note that $\alpha^0_{kk'} \leq 1$. The fourth line follows

$$\sum_{k' = k+1}^N Q_{kk'} v_1(k') \leq v_1(k + 1) \sum_{k' = k+1}^N Q_{kk'} = v_1(k + 1)(1 - Q_{kk}).$$

13Suppose there exists a $k$ such that $v_j(k) < v_j(k + 1)$ for $j = 0, 1$. This means that in the economy starting with one more defector, any defector is more likely to meet a cooperator. But this cannot be true, as it contradicts the properties of the transition matrix $Q$ as in Camera and Gioffré (forthcoming, Theorem 1).
Since

\[ v_1(k+1) \geq \frac{1 - \beta Q_{kk}}{\beta(1 - \beta Q_{kk})} v_0(k) - \frac{d}{\beta(1 - Q_{kk})}, \]

one gets

\[ v_1(k) - v_0(k-1) \geq \frac{1 - \beta Q_{k-1,k-1}}{\beta(1 - Q_{k-1,k-1})} v_0(k-1) - \frac{d}{\beta(1 - Q_{k-1,k-1})} v_0(k-1) \]

\[ = \frac{1 - \beta}{\beta(1 - Q_{k-1,k-1})} v_0(k-1) - \frac{d}{\beta(1 - Q_{k-1,k-1})} \]

\[ \geq \frac{1 - \beta}{\beta(1 - Q_{k-1,k-1})} \frac{d}{1 - \beta} - \frac{d}{\beta(1 - Q_{k-1,k-1})} = 0. \]

The last line follows from \( v_0(k) \geq v_0(N) = \frac{d}{1 - \beta} \) for all \( k = 2, \ldots, N \).

\[ \square \]

**Proof of Lemma 6.** Consider inequality (9). Derive an expression for \( v_1(k-1) - v_1(k) \) when \( k = 2, \ldots, N \). From the definition of \( v_1(k) \) in (5), for all \( k = 2, \ldots, N \),

\[ v_1(k-1) - v_1(k) = (g - d) \frac{\phi_{k-1} - \phi_k}{1 - \beta}. \]  

(10)

By Camera and Gioffrè (forthcoming, Theorem 3) it holds that \( \lim_{\beta \to 1 -} \frac{\phi_k}{1 - \beta} < \infty \). Hence, (9) holds whenever \( l \) is sufficiently large. To ensure that the parameter set is nonempty a second step is taken.

By Camera and Gioffrè (forthcoming, Theorem 3) it holds that \( \phi_k - \phi_{k+1} \leq \phi_{k-1} - \phi_k \) for all \( k = 2, \ldots, N \). So,

\[ v_1(k-1) - v_1(k) \leq v_1(1) - v_1(2) = (g - d) \frac{\phi_1 - \phi_2}{1 - \beta}. \]

We wish to prove that \( \frac{\beta(\phi_1 - \phi_2)}{1 - \beta} < 1 \), hence a value \( l^* < g - d \) exists, which satisfies
From equation (5) in Lemma 2 one obtains

\[ v_1(1) = \frac{1}{1 - \beta} [\phi_1 g + (1 - \phi_1) d]. \]

By definition of \( v_1(1) \), it also holds that

\[
v_1(1) = \sigma_1 g + (1 - \sigma_1) d + \beta v_1(2) = \sigma_1 g + (1 - \sigma_1) d + \frac{\beta}{1 - \beta} [\phi_2 g + (1 - \phi_2) d],
\]

where equation (5) for \( v_1(2) \) has been used. Hence, it must hold that

\[
\frac{1}{1 - \beta} [\phi_1 g + (1 - \phi_1) d] = \sigma_1 g + (1 - \sigma_1) d + \frac{\beta}{1 - \beta} [\phi_2 g + (1 - \phi_2) d],
\]

which is rewritten as

\[
\frac{\phi_1 - \beta \phi_2}{1 - \beta} = \sigma_1.
\]

Recall that \( \sigma_1 = 1 \) (if there is only one defector, then the defector meets a cooperator with certainty) and \( \beta (\phi_1 - \phi_2) < \phi_1 - \beta \phi_2 \), hence \( \frac{\beta (\phi_1 - \phi_2)}{1 - \beta} < 1 \).

Finally, note that inequality (9) is the most stringent when \( k = 2 \), and in this case it can be rewritten as

\[
(g - d) \frac{1}{m_0} \frac{\beta}{1 - \beta} \sum_{k' = 2}^{N} Q_{2k'} \alpha_{2k'}^0 (\phi_{k' - 1} - \phi_{k'}) \leq l,
\]

which is achieved by substituting on the right hand side \( \sum_{k' = 2}^{N} Q_{2k'} \mu_{2k'}^0 = m_0 \) and by substituting in the difference \( v_1(k' - 1) - v_1(k') \) the expressions \( v_1(k) \) given in Lemma 2.
Now define

$$\gamma := \sup_{\beta \in (0, 1)} \sum_{k'=2}^{N} Q_{2k'} \phi_{2k'}^0 \frac{\beta}{1 - \beta} (\phi_{k'-1} - \phi_{k'}).$$

Note that $\gamma < m_0$ because $\frac{\beta(\phi_{k'-1} - \phi_{k'})}{1 - \beta} < 1$ for all $\beta$. Hence, defining

$$l^* := (g - d) \gamma / m_0,$$

inequality (9) holds for all $\beta \in (0, 1)$ if $l$ is in $[l^*, g - d)$.

\[\square\]

**Proof of Proposition 2.** Consider gift-giving equilibrium. The transition matrix off equilibrium—call this matrix $P$—exhibits the same properties as matrix $Q$.\(^{14}\)

Similarly to what was done for the case of monetary trade, the off-equilibrium payoff of defector $i$, when there are $k$ defectors, is

$$v_g(k) = \frac{1}{1 - \beta} \left[ \phi_k^P g + (1 - \phi_k^P) d \right], \quad \text{for } k = 1, \ldots, N,$$

where

$$\phi_k^P := (1 - \beta) e_k^T (I - \beta P)^{-1} \sigma^P, \quad \text{for } k = 1, \ldots, N, \quad (11)$$

and $\sigma_k^P \in \sigma^P$ defines the probability that defector $i$ meets someone who makes a gift (a seller who cooperates), given $k$ defectors. As before, $\lim \frac{\phi_k^P}{1 - \beta} < \infty$. In addition, $\sigma_k^P \geq \sigma_k$ for all $k$ because every active seller makes a transfer; instead, under monetary

\(^{14}\)One can prove that, for $k' = k + h \geq k$, it has elements

$$P_{k,k+h} := \frac{1}{(N-1)!} \sum_{j=0}^{k'} \frac{k}{j} \left( \begin{array}{c} N-k \\ j \end{array} \right) (k-j-1)!!(N-k-j-1)!! \left( \begin{array}{c} j \\ h \end{array} \right) \left( \frac{1}{2} \right)^j$$

where $J_k = \min(k, N-k)$. For $k' < k$, the matrix elements are zero. The generic element $P_{k,k'}$ is the probability to go from $k$ defectors today to $k'$ tomorrow.
trade this happened only when the seller had no money.

Now fix \( M = 1 \) and suppose the player who has the token is a defector. Since there is only one token in the economy, it follows that \( \phi^p_k \leq \phi_k \). This can be easily proved because \( \sigma^p_k = \sigma_k \) when \( M = 1 \) and only the defector with the token can generate new defectors. In short, the contagious process works more slowly under monetary equilibrium, than under gift-giving, when \( M = 1 \). It follows that for some values of money supply, say \( M \leq \tilde{M} \) with \( \tilde{M} \geq 1 \), it holds that \( \phi^p_k \leq \phi_k \), i.e., the rate at which defectors can earn surplus over the long haul is greater under \( Q \) (monetary trade) than under \( P \) (gift-giving).

Now, consider a deviation in equilibrium. Under gift-giving, only the seller can deviate (buyers have no action to take), a deviation which is suboptimal if \( \beta [v_g - v_g(2)] \geq l \), which, using the definition of \( v_g(2) \), gives

\[
l \leq \beta \left[ \frac{(1 - \beta)v_g - d}{1 - \beta} - \frac{g - d}{1 - \beta} \phi^p \right].
\]

By continuity \( \beta_g \in (0, 1) \) exists such that \( \beta [v_g - v_g(2)] \geq l \) for \( \beta \in [\beta_g, 1) \). Under monetary trade, the equivalent condition is

\[
l \leq \beta \left[ \frac{(1 - \beta)v_1 - d}{1 - \beta} - \frac{g - d}{1 - \beta} \phi^p \right].
\]

Let \( M \leq \tilde{M} \); since \( v_1 \leq v_g \) and \( \phi^p_k \leq \phi_k \) it holds that \( \beta_g \leq \beta^* \). Hence, if \( M \leq \tilde{M} \) and \( \beta \in [\beta_g, \beta^*) \), then gift-giving equilibrium exists but monetary equilibrium does not. □

**Proof of Proposition 3.** Consider a large economy, i.e., \( N \to \infty \). Here, norms of voluntary behavior can only sustain the payoff \( \frac{d}{1 - \beta} \). Now introduce tokens. Let \( b \) denote the portion of agents who have one token, i.e., \( M = Nb \) as \( N \to \infty \). Suppose that society can adopt a technology for external enforcement of spot monetary trades (=
If each player invests \( c \geq 0 \) resources, on the initial date. A player is willing to sustain this cost, ex-ante, if

\[
\frac{d}{1 - \beta} \leq bv_1 + (1-b)v_0 - c
\]

Using (1) to substitute for \( v_0 \) and \( v_1 \), and noting that as \( N \to \infty \), \( m_0 = \frac{b}{2}, 1 - m_1 = \frac{1-b}{2} \), the inequality above becomes

\[
c(1-\beta) \leq \frac{(1-b)b}{2}(g-d-l).
\]

One can rewrite it as \( \beta \geq \beta_c \) where \( \beta_c := 1 - (1-b)b\frac{g-d-l}{2c} \), which increases in \( c \), converges to 1 when \( b \to 1 \) and \( b \to 0 \), and reaches a minimum when \( b = \frac{1}{2} \). From Proposition 1, \( \beta \geq \beta_m \) is needed. Hence, monetary equilibrium with costly external enforcement requires \( \beta \geq \max(\beta_m, \beta_c) \). If the cost is sufficiently small then \( \beta_c \leq \beta_m \), so introducing an explicit enforcement cost does not alter the equilibrium set; specifically, \( c \leq \frac{b}{2}[(1-b)(g-d) + (1+b)l] \) is needed. For costs larger than this value, monetary equilibrium exists only on the restricted subset \( [\beta_c, 1) \).
References


