Appendix–Not for Publication

Hence we may have a different number of homogeneous workers $i \in \mathcal{I}_t := \{1, 2, \ldots, x_t\}$ in each period t. x_t is realized before posting wages. Let $\mathcal{M}, \mathcal{H} : [0,1] \times [2, \infty) \to [0,1]$

$$\mathcal{M}(\pi, x_t) = 1 - (1-\pi)^{x_t}, \qquad \mathcal{H}(\pi, x_t) = \frac{\mathcal{M}(\pi, x_t)}{x_t \pi}.$$

Clearly, when $x_t \geq 2$ is an integer value, $\mathcal{M}(\pi, x_t), \mathcal{H}(\pi, x_t)$ are probabilities of trade in symmetric equilibrium as defined in the previous section.

The payoff of a firm in period t is

$$\Pi_t(x_t) = \max_v \{\mathcal{M}(\pi, x_t)\phi(v) + \beta \mathbb{E} \Pi_{t+1}(x_{t+1})\} = \max_v \{\mathcal{M}(\pi, x_t)\phi(v)\} + \beta \mathbb{E} \Pi_{t+1}(x_{t+1}),$$

where the expectation $\mathbb{E}$ is taken over x_{t+1}. The worker's strategy π must satisfy the indifference condition

$$\mathcal{H}(\pi, x_t) v = \mathcal{H}(\pi_j, x_t) v_j \qquad \text{for any } j \in \mathcal{J}.$$

We have some natural questions. What is the effect on the equilibrium wage v^* by a demand shock on x_t? How bit is the effect depending on $\bar{v}$ by a demand shock? From the previous analysis, we have the first order condition

$$\frac{\partial M(\pi(v^*, x_t), x_t)}{\partial \pi} \frac{\partial \pi(v^*, x_t)}{\partial v} (\bar{v} - v^*) - M(\pi(v^*, x_t), x_t) = 0.$$

$$\frac{\partial v^*}{\partial k} = -\frac{A(v^*, x_t)}{B(v^*, x_t)}$$

where

$$\begin{aligned} A(v^*, x_t) :=& \frac{\partial M(\pi(v^*, x_t), x_t)}{\partial \pi} \frac{\partial \pi(v^*, x_t)}{\partial v} > 0 \\ B(v^*, x_t) :=& \Big\{ \frac{\partial^2 M(\pi(v^*, x_t), x_t)}{\partial \pi^2} \Big(\frac{\partial \pi(v^*, x_t)}{\partial v} \Big)^2 + \frac{\partial M(\pi(v^*, x_t), x_t)}{\partial \pi} \frac{\partial^2 \pi(v^*, x_t)}{\partial v^2} \Big\} (\bar{v} - v^*) \\ & - 2 \frac{\partial M(\pi(v^*, x_t), x_t)}{\partial \pi} \frac{\partial \pi(v^*, x_t)}{\partial v} < 0 \end{aligned}$$

We denote

$$M := \mathcal{M}(\pi, x_t) = 1 - (1-\pi)^{x_t} \geq 0 \qquad M_\pi := \frac{\partial \mathcal{M}(\pi, x_t)}{\partial \pi} = x_t(1-\pi)^{x_t - 1} \geq 0$$
$$M_{x_t} := \frac{\partial \mathcal{M}(\pi, x_t)}{\partial x_t} = -(1-\pi)^{x_t} \ln(1-\pi) \geq 0$$
$$M_{\pi\pi} := \frac{\partial^2 \mathcal{M}(\pi, x_t)}{\partial \pi^2} = -x_t(x_t - 1)(1-\pi)^{x_t - 2} \leq 0$$
$$M_{\pi x_t} := \frac{\partial^2 \mathcal{M}(\pi, x_t)}{\partial x_t \, \partial \pi} = (1-\pi)^{x_t - 1}(1 + x_t \ln(1-\pi))$$
$$M_{\pi\pi x_t} := \frac{\partial^3 \mathcal{M}(\pi, x_t)}{\partial x_t \, \partial \pi^2} = -(1-\pi)^{x_t - 2}(2x_t - 1 + x_t(x_t - 1)\ln(1-\pi))$$
$$\pi_v := \frac{\partial \pi(v)}{\partial v} \geq 0 \qquad \pi_{vv} := \frac{\partial^2 \pi(v)}{\partial v^2} \leq 0$$
$$\varphi := \phi_{t+1,H}(v) + k \qquad \varphi_v := \frac{\partial(\phi_{t+1,H}(v) + k)}{\partial v} \qquad \varphi_{vv} := \frac{\partial^2(\phi_{t+1,H}(v) + k)}{\partial v^2}$$

$$M_{\pi x_t} \begin{cases} \geq 0 & \text{, if } x_t \leq -\dfrac{1}{\ln(1-\pi)} \\ < 0 & \text{, otherwise} \end{cases}$$
$$M_{\pi\pi x_t} \begin{cases} \geq 0 & \text{, if } x_t \leq -\dfrac{1}{\ln(1-\pi)} + \dfrac{1 + \sqrt{1 + \dfrac{4}{(\ln(1-\pi))^2}}}{2} \\ < 0 & \text{, otherwise} \end{cases}$$

$$\frac{\partial^2 v^*}{\partial k \, \partial x_t} = \frac{\partial^2 v^*}{\partial x_t \, \partial k}$$
$$= -\frac{(\dfrac{\partial A(v^*, x_t)}{\partial v}\dfrac{\partial v^*(x_t)}{\partial x_t} + \dfrac{\partial A(v^*, x_t)}{\partial x_t})B(v^*, x_t) - (\dfrac{\partial B(v^*, x_t)}{\partial v}\dfrac{\partial v^*(x_t)}{\partial x_t} + \dfrac{\partial B(v^*, x_t)}{\partial x_t})A(v^*, x_t)}{(B(v^*, x_t))^2}$$

Because

$$\frac{\partial^2 v^*}{\partial k \, \partial v} = 0 = \frac{\partial^2 v^*}{\partial v \, \partial k} = -\frac{\dfrac{\partial A(v^*, x_t)}{\partial v}B(v^*, x_t) - \dfrac{\partial B(v^*, x_t)}{\partial v}A(v^*, x_t)}{(B(v^*, x_t))^2}$$

we have

$$
\begin{aligned}
\frac{\partial^2 v^*}{\partial x_t\,\partial k} &= -\frac{\dfrac{\partial A(v^*,x_t)}{\partial x_t}B(v^*,x_t) - \dfrac{\partial B(v^*,x_t)}{\partial x_t}A(v^*,x_t)}{(B(v^*,x_t))^2} \\
&= -\frac{M_{\pi x_t}\pi_v(M_{\pi\pi}\pi_v^2\varphi + M_\pi\pi_{vv}\varphi + 2M_\pi\pi_v\varphi_v + M\varphi_{vv})}{(B(v,x_t))^2} \\
&\quad + \frac{M_\pi\pi_v(M_{\pi\pi x_t}\pi_v^2\varphi + M_{\pi x_t}\pi_{vv}\varphi + 2M_{\pi x_t}\pi_v\varphi_v + M_{x_t}\varphi_{vv})}{(B(v,x_t))^2} \\
&= -\frac{(M_{\pi x_t}M_{\pi\pi} - M_\pi M_{\pi\pi x_t})\pi_v^3\varphi + (M_{\pi x_t}M - M_\pi M_{x_t})\pi_v\varphi_{vv}}{(B(v,x_t))^2}
\end{aligned}
$$

Note that

$$
\begin{aligned}
&M_{\pi x_t}M_{\pi\pi} - M_\pi M_{\pi\pi x_t} \\
&\quad = -(1-\pi)^{x_t-1}(1 + x_t\ln(1-\pi))x_t(x_t-1)(1-\pi)^{x_t-2} \\
&\quad + x_t(1-\pi)^{x_t-1}(1-\pi)^{x_t-2}(2x_t - 1 + x_t(x_t-1)\ln(1-\pi)) \\
&\quad = x_t(x_t-2)(1-\pi)^{2x_t-3} \geq 0
\end{aligned}
$$

Hence $\dfrac{\partial^2 v}{\partial k\,\partial x_t} = \dfrac{\partial^2 v}{\partial x_t\,\partial k} \leq 0$, therefore

$$
\frac{\partial\Big(-\dfrac{\partial v}{\partial x_t}\Big)}{\partial k} = -\frac{\partial^2 v}{\partial k\,\partial x_t} \geq 0
$$

The change of wages based on the demand shocks are more sensitive with large k.