BOOK OF ABSTRACTS FOR THE CONFERENCE

# ADVANCES IN OPERATOR THEORY WITH APPLICATIONS TO MATHEMATICAL PHYSICS, CHAPMAN 2022

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ORGANIZERS: DRS DANIEL ALPAY, JUSSI BEHRNDT, FABRIZIO COLOMBO AND PRESIDENT DANIELE STRUPPA.

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**The Kaczmarz Algorithm in Banach Space**

*Dr. Anna Aboud,*
*Westmont College,*
*Santa Barbara, California*

The Kaczmarz algorithm reconstructs vectors in a Hilbert space using inner products against an *effective sequence*. The algorithm can be nontrivially expanded to reconstruct vectors within a Banach space, $X$. Starting with the mixed Gram matrix, we give necessary and sufficient conditions for a sequence to be effective in $X$. We discuss the relation of these conditions to the boundedness of an infinite product of associated operators.

**Discrete analytic functions, structured matrices and a new family of moment problems**

*Dr. Daniel Alpay*
*Chapman University,*
*Orange, California*

Using Zeilberger generating function formula for the values of a discrete analytic function in a quadrant we make connections with the theory of structured reproducing kernel spaces, structured matrices and a generalized moment problem. An important role is played by a Krein space realization result of Dijksma, Langer and de Snoo for functions analytic in a neighborhood of the origin. A key observation is that, using a simple Moebius transform, one can reduce the study of discrete analytic functions in the upper right quadrant to problems of function theory in the open unit disk. As an example, we associate to each finite positive measure on $[0, 2\pi]$ a discrete analytic function on the right-upper quarter plane with values on the lattice defining a positive definite function. Emphasis is put on the rational case, both when an underlying Carathéodory function is rational and when, in the positive case, the spectral function is rational. The rational case and the general case are linked via the existence of a unitary dilation, possibly in a Krein space.

This is joint work with Fabrizio Colombo, Kamal Diki, Irene Sabadini and Dan Volok.

**On the Mittag Leffler Bargmann (MLB) transform**

*Natanael Alpay*
*University of California at Irvine,*
*Irvine, California*

In this talk we introduce a Segal-Bargmann type transform associated to the Mittag Leffler Fock space introduced in (Rosenfeld et al. 2018). First, we study some basic properties of this operator and investigate how this can be connected to the classical Fourier transform. Then, we discuss the counterpart of the creation and annihilation operators in this setting. Finally, we present an extension of these results in the case of quaternions, in particular in the slice hyperholomorphic setting. This work is based on a joint work with Kamal Diki.

**Modification of the Christoffel Formula**

*Rachel Bailey*
*University of Connecticut,*
*Storrs, Connecticut*

It is well known that for a family of orthogonal polynomials with respect to a measure $\mu$, one can multiply the measure by a polynomial to obtain a new sequence of orthogonal polynomials via the Christoffel transformation. However, in some nonstandard situations, the Christoffel transformation cannot be directly applied. One of such situations is when the family of polynomials is a sequence of exceptional orthogonal polynomials. In this talk we will demonstrate why and how to modify the Christoffel transformation so it can be applied to one of the first known examples of exceptional orthogonal polynomials introduced by Dubov, Eleonskii, and Kulagin. In addition, it will be shown that the construction is generic and leads to a new class of exceptional orthogonal polynomials.

**A quantized version of the Jordan decomposition theorem for Borel measures**

*Dr. Joseph Ball,*
*Virginia Tech,*
*Blacksburg, Virginia*

Avector lattice is a real vector space equipped with a partial order $\leq$ which is compatible with the vector space structure ($x \leq y$ in $V \Rightarrow x + z \leq y + z$ for all $z \in V$, and $\alpha \geq 0$ in $\mathbb{R}$, $x \geq 0$ in $V \Rightarrow \alpha \cdot x \geq 0$ in $V$). Examples are $\mathbb{R}^{\Omega}$ (real-valued functions on a point set $\Omega$, $C_{\mathbb{R}}(\Omega)$ (continuous real-valued functions on a topological space $\Omega$, the $L^p$ spaces $L^p\mathbb{R}(\Omega, \Sigma, \mu)$, all with the pointwise ordering, $\mathcal{L}_{\rm sa}(\mathcal{H})$ (self-adjoint operators on a Hilbert space $\mathcal{H}$ with the Loewner ordering), finite signed measures $\mathcal{M}_{\mathbb{R}}(\Omega)$ with $\mu \geq 0$ meaning that $\mu(X) \geq 0$ for all measurablel subsets of $\Omega$, $\mathcal{K}_{\rm Her}(\Omega)$ (the space of operator-valued functions $K \colon \Omega \times \Omega \to \mathcal{L}(\mathcal{Y})$ such that $K$ is *Hermitian* meaning that $K^*(z, w) = K(w, z)^*$ with $K \geq 0$ where $K \geq 0$ means that $\sum_{i,j=1}^{N} \langle K(z_i, z_j) y_j, y_i \rangle \geq 0$ for all choices of $z_1, \ldots, z_N \in \Omega$, $y_1, \ldots, y_N \in \mathcal{Y}$). In all these examples except for the cases $C_{\mathbb{R}}(\Omega)$ and $K_{\rm Her}(\Omega)$ (unless $\Omega$ is finite), there is a Jordan Decomposition Theorem: any $x \in V$ can be decomposed as $x = x^+ - x^-$ where $x^+ = x \vee 0 \geq 0$ and $x^- = (-x) \vee 0 \geq 0$ (so $V$ is a *Riesz space*: any two elements $x, y$ have a least upper bound $x \vee y$). We discuss extensions of these ideas to a quantized/noncommutative settings involving e.g. linear maps between $C^*$-algebras or operator systems (unital, linear, self-adjoint subspaces of a $C^*$-algebra), as well as noncommutative functions and noncommutative kernels (where the function argument involves square matrices over points as well as just points). This is joint work with Victor Vinnikov (Ben Gurion University) and Gregory Marx (recently of Virginia Tech).

## Sharp boundary trace theory and Schrödinger operators on bounded Lipschitz domains

*Dr. Jussi Behrndt*
*TU Graz,*
*Graz, Austria*

We develop a sharp boundary trace theory in arbitrary bounded Lipschitz domains which, in contrast to classical results, allows "forbidden" endpoints and permits the consideration of functions exhibiting very limited regularity. This is done at the (necessary) expense of stipulating an additional regularity condition involving the action of the Laplacian on the functions in question which, nonetheless, works perfectly with the Dirichlet and Neumann realizations. In turn, this boundary trace theory serves as a platform for developing a spectral theory for Schrödinger operators on bounded Lipschitz domains, along with their associated Weyl–Titchmarsh operators. This talk is based on joint work with Fritz Gesztesy and Marius Mitrea.

**Fixed points of generalized Schur functions and inequalities for derivatives at these points**

*Dr. Vladimir Bolotnikov*
*College of William & Mary,*
*Williamsburg, Virginia*

By the Schwarz-Pick and Denjoy-Wolff theorems, a Schur-class function $f$ (analytic and bounded by one in modulus on the open unit disk $\mathbb{D}$) either has a unique fixed point $\zeta_0 \in \mathbb{D}$ (and then necessarily $|f'(\zeta_0)| \leq 1$) or it has a unique boundary fixed point $\zeta_0 \in \mathbb{T} := \partial\mathbb{D}$ such that $f'(\zeta_0) \leq 1$. Besides this distinguished fixed point (the *Denjoy-Wolff* point), $f \in \mathcal{S}$ may have other boundary fixed points $\zeta \in \mathbb{T}$ with $f'(\zeta) > 1$. If $K \subset \mathbb{T}$ is the set of all such points, then necessarily $\sum_{\zeta \in K} \frac{1}{f'(\zeta) - 1} < \infty$. Upper bounds for the latter sum in terms of $f'(\zeta_0) \neq 1$ were established in [1]: the sum does not exceed $\frac{1 - |f'(\zeta_0)|^2}{|1 - f'(\zeta_0)|^2}$ if $\zeta_0 \in \mathbb{D}$ and $\frac{f'(\zeta_0)}{1 - f'(\zeta_0)}$ if $\zeta_0 \in \mathbb{T}$. The case where $f'(\zeta_0) = 1$ will be discussed in the talk as well as several extensions of the above results to the more general case where $f$ is meromorphic on $\mathbb{D}$ with $\kappa > 0$ poles and subject to the condition $\lim_{r \to 1^-} \sup_{|z|=r} |f(z)| \leq 1$; equvalently, $f$ is a coprime ratio of a Schur-class function and a finite Blaschke product of degree $\kappa$.

## Gauging the global phase of the qubit

*Luke Burns*
*Chapman University,*
*Orange, California*

After a short review of algebraic approaches to quantum mechanics, we propose a formalism for a virtual qubit system in which global phase symmetry is promoted to a local one in such a way that gauge-invariant states are represented instead by dynamical representatives of their gauge group, somewhat analogous to the representation of electromagnetic fields by gauge potentials in a manner to be made precise. We discuss a few basic algebraic results, as well as questions of operational significance and generalization to higher dimensional systems.

**Darboux transformations and matrix-valued exceptional orthogonal polynomials**

*Dr. Riley Casper*
*California State University at Fullerton,*
*Fullerton, California*

Exceptional orthogonal polynomials are sequences of polynomials satisfying a second order differential operator and spanning a finite codimensional subspace of the vector space of polynomials. Recently, these have been completely classified in terms of Darboux transformations of classical orthogonal polynomials. In this talk, we will introduce a matrix-valued analog of exceptional orthogonal polynomials and present a corresponding classification scheme in terms of noncommutative bispectral Darboux transformations.

## Generalized White noise analysis and stochastic distributions

*Dr. Paula Cerejeiras,*
*University of Aveiro,*
*Aveiro, Portugal*

**Padé approximations of Herglotz functions and applications to homogenization and composites**

*Dr. Elena Cherkaev,*
*University of Utah*
*Salt Lake City, Utah*

Pade approximations of Stieltjes and Herglotz-Nevanlinna functions play a central role in the operator theory connecting the moment problem with continuous fractions, Jacobi matrices, orthogonal polynomials, the spectral theory, and approximation problems. The talk will discuss matrix Pade approximations of the resolvents of operators arising in homogenization problem. Stieltjes/ Herglotz function integral representations for the homogenized transport coefficients of composites link composite microgeometry to the spectral properties of the related self-adjoint operator. I will show that a matrix spectral measure in the integral representation of the effective properties of composite can be uniquely reconstructed; this uniqueness provides a basis for the inverse homogenization problem of recovering information about the microgeometry of the medium. Pade approximations of the spectral measure allow to construct bounds on the effective properties of composites in forward homogenization and result in spectrally matched geometries in inverse homogenization linking it to the inverse spectral problem.

**The fine structure of the spectral theory on the S-spectrum in dimension five**

*Dr. Fabrizio Colombo*
*Politecnico di Milano,*
*Milano, Italy*

Holomorphic functions play a crucial role in operator theory and the Cauchy formula is a very important tool to define functions of operators. The Fueter-Sce-Qian extension theorem is a two steps procedure to extend holomorphic functions to the hyperholomorphic setting. The first step gives the class of slice hyperholomorphic functions; their Cauchy formula allows to define the so-called S-functional calculus for noncommuting operators based on the S-spectrum. In the second step this extension procedure generates monogenic functions; the related monogenic functional calculus, based on the monogenic spectrum, contains the Weyl functional calculus as a particular case. In this talk we show that the extension operator from slice hyperholomorphic functions to monogenic functions admits various possible factorizations that induce different function spaces. The integral representations in such spaces allow to define the associated functional calculi based on the S-spectrum. The function spaces and the associated functional calculi define the so called fine structure of the spectral theories on the S-spectrum. Among the possible fine structures there are the harmonic and poly-harmonic functions and the associated harmonic and poly-harmonic functional calculi.

**Thermodynamic expansions via perturbation theory, and the dimension theory of continued fraction Cantor sets**

*Dr. Tushar Das*
*University of Wisconsin La Crosse,*
*La Crosse, Wisconsin*

Continued fractions have provided a natural playground for several developments in number theory, analysis, dynamics, physics, and probability theory. We highlight some relatively recent results (2007.10554,1910.10259 on arXiv) obtained by leveraging the thermodynamic formalism à la Bowen–Sinai–Ruelle to the dimension theory of continued fraction Cantor sets – a rich research seam pioneered around ninety years ago by Jarnik, Besicovitch, Kurzweil, and Good. The talk will be accessible to those interested in some convex combination of functional analysis, dynamical systems, fractal geometry, and number theory. Most importantly, I hope to present a sampling of open questions and research directions that await exploration.

**A polyanalytic functional calculus and on the $S$-spectrum**

*Dr. Antonino De Martino*
*Chapman University,*
*Orange, California*

The Fueter theorem provides a two step procedure to build an axially monogenic function, i.e. a null-solutions of the Cauchy-Riemann operator in $\mathbb{R}^4$, denoted by $\mathcal{D}$. In the first step a holomorphic function is extended to a slice hyperholomorphic function, by means of the so-called slice operator. In the second step a monogenic function is built by applying the Laplace operator in four real variables ($\Delta$) to the slice hyperholomorphic function. In this talk we use the factorization of the Laplace operator, i.e. $\Delta = \overline{\mathcal{D}}\mathcal{D}$ to split the previous procedure. From this splitting we get a class of functions that lies between the set of slice hyperholomorphic functions and the set of axially monogenic functions: the set of axially polyanalytic functions of order 2, i.e. null-solutions of $\mathcal{D}^2$. We show an integral representation formula for this kind of functions. The formula obtained is fundamental to define the associated functional calculus on the $S$-spectrum. Moreover, in this talk we will show some properties of this polyanalytic functional calculus of order 2 like the resolvent equation and the product rule.

## Perfect quantum state transfer on diamond fractal graphs

*Dr. Maxim Derevyagin*
*University of Connecticut,*
*Storrs, Connecticut*

In the quest for designing novel protocols for quantum information and quantum computation, an important goal is to achieve perfect quantum state transfer for systems beyond the well-known one-dimensional cases, such as 1D spin chains. In this talk, I will review a series of recent papers where we introduced a method that allows to construct a new class of quantum spin chains on fractal-like graphs.

## Polyanalytic functions of infinite order: Fock and Hardy cases

*Dr. Kamal Diki*
*Chapman University,*
*Orange, California*

In this talk we introduce and study some reproducing kernel Hilbert spaces of polyanalytic functions of infinite order. As a first example we consider the counterpart of the Fock space in this framework whose kernel function is given by $e^{z\overline{w}+\overline{z}w}$. We show that this poly-Fock kernel of infinite order can be expressed in terms of a power series involving poly-Fock kernels of finite order. Then, we prove new results related to Segal-Bargmann and Berezin type integral transforms in this setting and discuss Landau levels operator. If time allows we will consider also the reproducing kernel Hilbert spaces of complex-valued functions with reproducing kernel $\dfrac{1}{(1-z\overline{w})(1-\overline{z}w)}$ and $\dfrac{1}{1-2\mathrm{Re}\, z\overline{w}}$. The corresponding backward shift operators will be studied. This talk is based on a joint work with Daniel Alpay, Fabrizio Colombo and Irene Sabadini.

**Spin Angular Momentum in Acoustic Field Theory**

*Dr Justin Dressel*
*Chapman University,*
*Orange, California*

Using an acoustic analog of Minkowski geometry, we construct a Lagrangian representation of acoustic field theory that accounts for the recently measured nonzero spin-angular-momentum density of sound fields in fluids or gases. While the traditional acoustic Lagrangian representation of the measured pressure and velocity fields using a dynamical scalar potential is unable to describe the vector character of the spin, we show that the pressure-velocity 4-vector additionally admits a dynamical bivector potential that correctly accounts for the spin. In the equilibrium frame this bivector potential reduces to a displacement field with amplitude equal to the scalar potential, such that its gauge freedom is equivalent to the arbitrary choice of displacement origin. The two potentials combine into an even-graded spinor potential with dynamics that recover the observed local radiation forces and torques on small probe particles, as well as the correct canonically conserved momentum and spin densities as proper Noether currents. This twin-potential construction for acoustics closely mirrors a formulation of vacuum electromagnetism that combines both electric- and magnetic-potential representations into a manifestly dual-symmetric odd-graded potential.

## Magnetic transport in quantum layers

*Dr. Pavel Exner*
*Doppler Institute for Mathematical Physics,*
*and Applied Mathematics, Prague, Czech Republic*

It is well known that while a homogeneous magnetic field localizes motion of a charged particle in the perpendicular direction, an infinitely long obstacle of various types, due to a potential or a variation of the field, can give rise to an absolutely continuous spectrum of the corresponding Hamiltonian. In this talk we discuss a different, purely geometric way to produce such a magnetic transport. We consider a particle confined to a layer with Dirichlet boundary and show that an absolutely continuous spectrum appears when the layer is bent, or if the perturbation consists of opening a window in the common boundary of two adjacent layers; we also provide conditions under which a part of the point spectrum may survive. We note that in contrast to the cases of a potential or magnetic perturbation, the effect we discuss here has no classical counterpart.

**Entropy production in anisotropic temperature fields**

*Dr. Tryphon Georgiou*
*University of California at Irvine,*
*Irvine, California*

Stochastic Thermodynamics represents a rapidly developing framework where the tools of stochastic analysis and stochastic control have allowed quantitative predictions on the efficiency, power, and dissipation that one can expect from physical thermodynamic systems. In this work, we discuss a particular paradigm known as the Brownian gyrator, where anisotropy of temperature fields (or, equivalently, chemical potentials, ion concentration gradients and so on), generate torque and the conditions for power generation in microscopic and biological engines. Indeed, it is reasonably well accepted that anisotropy (thermal, chemical and so on) in the real world provides the fuel that feeds dynamical processes that sustain life.

Dynamical flows in driven by anisotropy incur losses manifested as entropy production. In this work we consider a rudimentary model of an over damped stochastic thermodynamic system in an anisotropic temperature heat bath, and analyze the entropy production while driving the system between thermodynamic states in finite time. It is noted that entropy production in a fully isotropic temperature field, can be expressed as the Wasserstein $W_2$ length of the path traversed by the thermodynamic state of the system. In the presence of an anisotropic temperature field, the mechanism of entropy production is substantially more complicated as, besides dissipation, it entails seepage of energy between the ambient heat sources by way of the system dynamics. We show that, in this case, the entropy production can be expressed as the solution of a suitably constrained and generalized Optimal Mass Transport (OMT) problem. In contrast to the situation in standard OMT, entropy production may not be identically zero, even when the thermodynamic state remains unchanged. Physically, this is due to the fact that maintaining a Non-Equilibrium Steady State (NESS), incurs an intrinsic entropic cost. As already noted, NESSs are the hallmark of life and living systems by necessity operate away from equilibrium. Thus our problem of minimizing entropy production appears of central importance in understanding biological processes, such as molecular motors and motor proteins, and on how such processes may have evolved to optimize for available usage of recourses.

Joint work with O. Movilla Miangolarra, A. Taghavaei.
OMM and TTG are with Mechanical and Aerospace Engineering, University of California, Irvine, CA 92697, USA AT is with Aeronautics and Astronautics, University of Washington, Seattle, WA 98195 USA

**Higher order correlations**

*Dr. Alberto Grünbaum*
*University of California at Berkeley,*
*Berkeley, California*

The possibility of changing a real valued function $f(x)$ without changing its correlation functions (given by integrals of products of $f(x)$ with arbitrary translates of it) appears in several areas, ranging from:

quantum mechanics
signal processing (Wiener's theory of nonlinear noise)
X-ray crystallography.

More recently it has played a role in cosmology in connection with the "cosmic background radiation" distribution.

I will give an ab-initio description of the problem in different setups and discuss similarities as well as differences that arise. The real problem is to come up with good reconstruction algorithms: finding $f(x)$ from the knowledge of some of these higher order correlations. This is a wide open area of research. Here are some references:

Wiener's theory of nonlinear noise, H.P. McKean pp 191-209 SIAM-AMS Proc. Vol VI 1973.

The square of a Gaussian process, F. A. Grünbaum Z. Wahrs. und Verw. Gebeite, 23, 1972 pp 121-124

The use of higher order invariants in the determinantion of generalized Patterson cyclotomic sets, F. A. Grünbaum and C. Moore, Acta Crystall. (1995) A51, pp 310-323

The Galaxy and Mass N-point correlation functions: a blast from the past , P.J.E. Peebles Historical developments in moderm cosmology ASP Conference series

## Perfect quantum strategies for XOR games

*Dr. Bill Helton*
*University of California at San Diego,*
*San Diego, California*

The talk will describe some of the structure associated with 'perfect strategies' for a class of cooperative games. In such problems one has a (noncommutative) algebra A which encodes quantum mechanical laws and a list of matrix equations . A solution to these amounts to a perfect quantum strategy; matrix solutions having size 1 is a perfect classical strategy.

The focus will be on 3XOR games and new results there. How likely is existence of a quantum solution in the absence of any classical solution, a key question, leads to a (seemingly new) class of random matrix problems. The talk will describe this structure and recent progress on the random matrix problem.

The work is joint with Adam Bene Watts ,Zehong Zhao, Jared Huges and Daniel Kane.

**Recent developments and publication opportunities in the journals and book programs at Birkhäuser.**

*Dr Jan Holland,*
*Birkäuser Science.*
*Basel, Switzerland*

We will discuss recent developments and publication opportunities in the journals and book programs at Birkhäuser (joint talk with Christopher Tominich).

**Quantum $f$-divergences via Nussbaum-Szkoła Distributions with applications to Petz-Rényi and von Neumann Relative Entropy**

*Dr. Tiju Cherian John*
*University of South Carolina,*
*Columbia, South Carolina*

Using an analysis of the spectral measure associated with the relative modular operator, we prove that the quantum $f$-divergence of two states (density operators on a Hilbert space) is same as the classical $f$-divergence of the corresponding Nussbaum-Szkoła distributions. This provides a general framework to study most of the known quantum entropic quantities using the corresponding classical entities. In this spirit, we study the Petz-Rényi and von Neumann relative entropy and prove that these quantum entropies are equal to the corresponding classical counterparts of the Nussbaum-Szkoła distributions. This is a generalization of a finite dimensional result that was proved by Nussbaum and Szkoła [Ann. Statist. 37, 2009, 2] to the infinite dimensions. We apply classical results about Rényi and Kullback-Leibler divergences to obtain new results and new proofs for some known results about the quantum relative entropies. Most important among these are (i) a quantum Pinsker type inequality in the infinite dimensions, and (ii) necessary and sufficient conditions for the finiteness of the Petz-Rényi $\alpha$-relative entropy for any order $\alpha \in [0, \infty]$. Furthermore, we construct an example to show that the information theoretic definition of the von Neumann relative entropy is different from Araki's definition of relative entropy when the dimension of the Hilbert space is infinite. This discrepancy can be bridged using the notion of the distribution of an unbounded positive selfadjoint operator with respect to a positive compact operator and Haagerup's extension of the trace. Our results are valid in both finite and infinite dimensions and hence can be applied to continuous variable systems as well.

## Generalized super-phenomena for arbitrary quantum observables

*Dr. Andrew Jordan,*
*Chapman University,*
*Orange, California*

I will show how to generalize superoscillations to arbitrary observables in quantum mechanics. Super phenomena of total angular momentum and energy will be described. Using the example of a sequence of harmonic oscillators, I will demonstrate that high energy approximate solutions can be created from asymptotically zero energy solutions that converges everywhere on the real line in a certain mathematical limit. These results can be applied to the spectral analysis of arbitrary differential equations.

### Quaternionic non-selfadjoint operators and their spectral theory

*Dr Uwe Kaehler, University of Aveiro,*
*Aveiro, Portugal*

**On weak/strong attractor for a $3-D$ Structural Acoustic Interaction with an Interface**

*Dr. Irena Lasiecka*
*University of Memphis,*
*Memphis, Tennessee*

$3-D$ structural acoustic interaction with an elastic wall modeled by a nonlinear Kirchhoff-Boussinesq dynamic plate equation is considered. The model arises within a context of noise suppression/control in an acoustic chamber surrounded by rigid and elastic walls. The attenuation/control of the noise takes place on an elastic wall which by itself is modeled by plate equation with supercritical restoring forces and Boussinesq nonliear term leading o a potential finite time blow up of solutions. In addition to Boussinesq source [resulting from a strong nonlinear coupling between vertical and transverse shear effects in the Midlin Timoshenko system], plate dynamics is ignited by an acoustic pressure resulting from wave propagation in an acoustic chamber. The key to the result is a boundary feedback control applied on the elastic wall. Hadamard wellposedness of weak and strong solutions will be discussed along with the long time behavior of the corresponding solutions. From mathematical point of view, the interesting features are the lack of dissipativity and the lack of compactness resulting from nonlinear plate effects and lack of time reversibility resulting from boundary control of acoustic waves. As it is known these features are fundamental in the theory of dynamical systems arising in hyperbolic like dynamics. To contend with the difficulty, a new "hybrid" method suitable for handling long time behavior of coupled dynamics with "poor communication" between the components will be presented.

**Quantitatively Hyper-Positive Rational Functions**

*Dr. Izchak Lewkowicz*
*Ben-Gurion University of the Negev,*
*Beer-Sheva, Israel*

In scalar terminology, *Positive* (real) functions map the right half plane, to its closure. Recently, we introduced the subset of *quantitatively, Hyper-Positive* (real) functions: Mapping the right half plane into disks within the right half plane. (Under inversion, these disks are mapped onto themselves) In engineering literature, quantitatively Hyper-Positive real rational functions, have existed, implicitely. The radius of the disk a family of Hyper-Positive (real) functions blong to, induces a partial order among these functions. At the limit, one recovers the classical set of positive real functions. As time permits, in this talk we explore properties of these (matrix-valued) rational functions.

Joint work with Daniel Alpay, Chapman University, California

## Poly slice monogenic functions and the PS-functional calculus

*Dr. Irene Sabadini Politecnico di Milano,*
*Milano, Italy*

The spectral theory based on the $S$-spectrum has been widely studied in the past fifteen years and has applications, for example, in the formulation of quaternionic quantum mechanics, in Schur analysis and in fractional diffusion problems. In this talk we introduce the theory of poly slice monogenic functions and the associated functional calculus, called $PS$-functional calculus, which is the polyanalytic version of the $S$-functional calculus and which is based on the notion of $S$-spectrum. We study the formulation of the calculus and we discuss some of its properties.

The talk is based on joint work with D. Alpay, F. Colombo, K. Diki.

## Pseudodifferential operators on the circle

*Dr. Ahmed Sebbar, Chapman University*
*Orange, California*

We show how the classical polylogarithm function $\mathrm{Li}_s(z)$ and its relatives, The Hurwitz Zeta function, the Lerch function can explain many properties of the complex powers of the Laplacian on the circle and of the distribution $(x + i0)^s$.

**Recent developments and publication opportunities in the journals and book programs at Birkhäuser.**

*Christopher Tominich,*
*Birkäuser/ Springer,*
*New-York, New-York*

We will discuss recent developments and publication opportunities in the journals and book programs at Birkhäuser (joint talk with Dr Jan Holland).

**Unique Continuation Properties of static eigen-problems: the ignition keys for Uniform Stabilization of Fluids by Feedback Controllers**

*Dr. Roberto Triggiani*
*University of Memphis,*
*Memphis, Tennessee*

In dealing with uniform stabilization of unstable parabolic problems, the first critical step is to ascertain Kalman's controllability condition of the projected finite dimensional unstable component [R.Triggiani, JMAA 1975]. To this end, Unique Continuation Properties for the adjoint problem are needed. Several UCPs are established in [R.Triggiani-X.Wan, AMO, 2021], by using arguments based on Carleman-type estimates of the Laplace operator, specialized from hyperbolic or Schrodinger Carleman-type estimates [Lasiecka-Triggiani-Zhang, early 2000]. These include the required UCPs for the localized interior as well as the localized boundary-based uniform stabilization of an unstable Boussinesq-system, following the case of the unstable Navier-Stokes equations [R.Triggiani, NonLinear Analysis, 2009].

## Hypertwined quantum field theory

*Dr. Adrian Vajiac*
*Chapman University*
*Orange, California*

Hypertwined analysis is a refinement of general hypercomplex theories of differential operators. In the assumption that the superspace has a hypertwined structure, I will discuss several (re)interpretations of notions and results in Quantum Field Theory (QFT), such as supersymmetry, path–integral quantization, cancellation of anomalies, et al. Particular QFTs of interest in this study are Topological QFTs such as supersymmetric Yang– Mills theory on a four–manifold, topological sigma models, and Chern–Simmons gauge theory, which are deeply related with Donaldson and Seiberg–Witten invariants, Gromov–Witten invariants, and knot and links invariants, respectively.

**Applications of Hypercomplex Analysis to Neural Networks**

*Dr. Mihaela Vajiac*
*Chapman University*
*Orange, California*

We discuss some recent developments in this directions. Work in collaboration with D. Alpay and K. Diki

**Fractional powers of nondivergence form elliptic operators**

*Dr. Mary C Vaughan*
*University of Texas at Austin,*
*Austin, Texas*

In this talk, we will begin by using the method of semigroups to define fractional powers of nondivergence form elliptic operators in bounded domains under minimal regularity assumptions. We will characterize a Poisson problem driven by such operators with a degenerate/singular extension problem. We then prove Harnack inequality for solutions to the extension equation which in turn implies Harnack inequality for solutions to the fractional Poisson problem. This is joint work with Pablo Raúl Stinga (Iowa State).

**A Paley-Wiener Type Theorem for Singular Measures on** $(-1/2, 1/2)$

*Dr. Eric Weber*
*Iowa State University,*
*Ames, Iowa*

For a fixed singular Borel probability measure $\mu$ on $(-1/2, 1/2)$, we give several characterizations of when an entire function is the Fourier transform of some $f \in L^2(\mu)$. The first characterization is given in terms of criteria for sampling functions of the form $\hat{f}$ when $f \in L^2(\mu)$. The second characterization is given in terms of criteria for interpolation of bounded sequences on $\mathbb{N}_0$ by $\hat{f}$. Both characterizations use the construction of Fourier series for $f \in L^2(\mu)$ demonstrated by Herr and Weber via the Kaczmarz algorithm and classical results concerning the Cauchy transform of $\mu$.

## Can one expect division/interpolation close-form formulae to be carried to the non-commutative setting ?

*Dr. Alain Yger*
*Université de Bordeaux,*
*Talence, France*

Multivariate residue theory reveals to be in the commutative setting a powerful tool to produce division/interpolation close formulae, for example in polynomial algebra (Hilbert's nullstellensatz, Briançon-Skoda-Huneke theorem) or in harmonic analysis, as explicit formulations of Ehrenpreis's-Palamodov Fundamental Principle for representation of solutions of systems of PDE's or even, under some conditions, mean-periodic functions satisfying systems of convolution equations with compactly supported convolutors. A major stumbling block is the crucial role played by commutativity. In this talk, I will discuss some ideas which arise from such calculus and could be transposed towards situations where one is facing non-commutativity. Quaternions, skew fields, offer such (limited) possibilities. I will also discuss some long-term work in progress with Daniel Alpay, which goal is a systematic introduction of Ore's (ore skew) calculus in attempts towards a formal modelization of non-stationarity.

**On Fractional Operators and their Applications in Physics and Engineering**

*Dr. Ahmed I. Zayed*
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Fractional operators having integral representations on $L^2(\mathbb{R}^n)$ play a vital role in physics and engineering. In this talk we shall discuss one of these operators and its action on the Wigner Distribution function which is a time-frequency representation of audio and video signals. The Wigner distribution is closely related to the radar ambiguity function which is a corner stone in radar theory. We will also shed light on other applications of fractional operators in optics and quantum mechanics and on related uncertainty principle.