BOOK OF ABSTRACTS FOR THE CONFERENCE

ADVANCES IN OPERATOR THEORY WITH APPLICATIONS TO MATHEMATICAL PHYSICS

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Internal and input/output system stability: the free noncommutative setting

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(joint work with Gilbert Groenewald and Sanne ter Horst)

Given a discrete-time linear system

$$\begin{cases} x(n+1) & = & Ax(n) + Bu(n) \\ y(n) & = & Cx(n) + Du(n), \end{cases}$$

internal exponential stability means that there exists numbers $M > 0$ and $0 < \rho < 1$ so that $\|x(n)\| \le M\rho^n$ for any system trajectory (u, x, y) with zero input signal ($u(n) = 0$ for all n), while *input/output stability* means that $\sum_{n=0}^{\infty} \|y(n)\|^2 \le \sum_{n=0}^{\infty} \|u(n)\|^2$ for any system trajectory (u, x, y) with $x(0) = 0$. For the finite-dimensional case, internal stability is characterized by the condition that the matrix A should have all eigenvalues in the unit disk, while input/output stability is characterized by the condition that there exist a positive-definite solution H to the Linear Matrix Inequality

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^* \begin{bmatrix} H & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} - \begin{bmatrix} H & 0 \\ 0 & I \end{bmatrix} \prec 0.$$

We indicate how these ideas extend to the setting of Structured Noncommutative Multidimensional Linear Systems having evolution along a free semigroup with d-generators (i.e., a rooted tree with d branches coming out of each node).

Inverse potential problems in divergence form and total variation regularization

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Based on joint work with: S. Chevillard, D. Hardin, J. Leblond, C. Villalobos-Guillen.

Inverse potential problems in divergence form consist in recovering a $\mathbb{R}^3$-valued measure on $\mathbb{R}^3$ knowing (one component of) the field of the Newton potential of its divergence on a piece of surface, away from the support. Such issues typically arise in source identification from field measurements for Maxwell's equations, in the quasi-static regime. They occur for instance in Electro-Encephalography, Magneto-Encephalography, Geomagnetism and Paleomagnetism, as well as in several non-destructive testing problems. A prototypical example, motivating the present study, is the problem of recovering an unknown magnetization distribution $\mu = (\mu_1, \mu_2, \mu_3)^t$, where the μ_j are finite signed measures on $\mathbb{R}^3$ supported in some closed set $S \subset \mathbb{R}^3$, from values of the magnetic field it generates on a closed set $Q \subset \mathbb{R}^3$ disjoint from S. By the Biot-Savard law, this field is the gradient of the Newton potential Φ_μ of the distribution $\operatorname{div}\mu$:

$$\Phi_\mu(x) := \frac{1}{4\pi}\int \frac{x-y}{|x-y|^3}\cdot d\mu(y), \qquad x \notin \operatorname{supp}\mu.$$

That is, Φ_μ is the solution to the Poisson equation $\Delta u = \operatorname{div}\mu$ which is "smallest at infinity".
We concentrate here on the case where S is compact and included in the horizontal plane $\mathbb{R}^2 \times \{0\} \sim \mathbb{R}^2$, while Q is compact and included in the horizontal plane $\mathbb{R}^2 \times \{h\}$ for some $h > 0$. We assume, to fix ideas, that $S = \overline{\Omega}$ with Ω a relatively compact Lipschitz-smooth open set, and that the vertical component of the magnetic field generated by μ is measured on Q. This correspond to the classical setup of Scanning Magnetic Microscopy for thin samples.
The so-called forward operator b_3 maps μ to the restriction of $\partial_{x_3}\Phi_\mu$ on Q. Here, ∂_{x_j} indicates the partial derivative with respect to the j-th coordinate x_j of $x \in \mathbb{R}^3$. If μ is seeked in the the form $m dx_1 dx_2$ with $m = (m_1, m_2, m_3)^t$ where $m_i \in L^2(S)$, we may regard b_3 as a compact operator $(L^2(S))^3 \to L^2(Q)$, given by

$$b_3[m] = -\frac{\mu_0}{2}\left(\partial_{x_1} P_h \star \widetilde{m}_1 + \partial_{x_2} P_h \star \widetilde{m}_2 + [\partial_{x_3} P_{x_3} \star \widetilde{m}_3]_{|x_3=h}\right)_{|Q}, \tag{0.1}$$

where P_h is the familiar Poisson kernel of the half-space at height $h > 0$ and $\widetilde{m}_i$ the extension of m_i by zero to all of $\mathbb{R}^2$. It can be shown that the kernel of b_3 consists of those m such that $m_3 = 0$ and $\operatorname{div}(\widetilde{m_1}, \widetilde{m}_2)^t = 0$, as a distribution on $\mathbb{R}^2$.
A classical (Tychonov) regularization scheme is then to look for:

$$\inf_{m\in[L^2(S)]^3} \|b_3[m] - d\|^2_{L^2(Q)} + \lambda\|m\|^2_{[L^2(S)]^3}, \tag{0.2}$$

where $d = b_3(m^*) + e$ is the measured data, corresponding to the field of the "true" magnetization $m^* dx_1 dx_2$ corrupted by some noise e, and $\lambda > 0$ is the so-called regularization parameter. It is standard that, as $\lambda \to 0$ and $\|e\|_{L^2(S)}/\lambda \to 0$, then the unique minimizer in (0.2) converges to the magnetization m^r equivalent to m^* (*i.e.* having the same image under b_3) of minimum L^2-norm on S. We show that this regularization scheme "spreads out" the recovery, in that if m^* is identically zero on an open subset of S, then m^r is not. We then consider the more general case where μ is a finite $\mathbb{R}^3$-valued measure supported on S, and we consider the regularization scheme:

$$\inf_{\mu \in [\mathcal{M}(S)]^3} \|b_3[\mu] - d\|^2_{L^2(Q)} + \lambda \|\mu\|_{TV}, \tag{0.3}$$

where $\mathcal{M}(S)$ is the space of finite signed measures supported on S and $b_3 : (\mathcal{M}(S))^3 \to L^2(Q)$ is as in (0.1) except that $\widetilde{m}_j dx_1 dx_2$ gets replaced by $d\mu_j$, while $\|\mu\|_{TV}$ indicates the total variation of μ. We show that when $\lambda \to 0$ and $\|e\|_{L^2(S)}/\sqrt{\lambda} \to 0$, and if the support of the "true magnetization" μ^* contains no loop, then each minimizer in (0.3) converges in the narrow sense to μ^*. Hence, this regularization scheme allows some "sparse recovery". The result generalizes to the case where μ^* has purely 1-unrectifiable support and S is a "sufficiently thin" set, not necessarily contained in a plane, and also to magnetizations supported in a plane which point in a fixed direction. This leads to "sparse recovery" results in this context, in the sense of narrow convergence of measures.

The Birman-Schwinger principle for generalized eigenvectors

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The Birman-Schwinger principle is one of the standard tools in spectral analysis of self-adjoint Schrödinger operators. This useful technique allows to reduce the eigenvalue problem for the Schrödinger operator $-\Delta + V$ to an eigenvalue problem involving a sandwiched resolvent of the unperturbed operator $-\Delta$. In this talk we first review the classical Birman-Schwinger principle and illustrate it with some typical applications in spectral analysis. Afterwards we discuss some recent extensions for the characterization of the generalized eigenvectors of non-selfadjoint Schrödinger operators and other general non-selfadjoint second order elliptic differential operators.

The talk is based on a joint work with A.F.M. ter Elst and F. Gesztesy.

Quasi-monogenic functions and Riesz-Hilbert transforms

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Clifford analysis is a higher dimensional function theory that generalizes the function theory of one complex variable. It is also a refinement of harmonic analysis. It involves the study of monogeic functions with values in a Clifford module. A monogenic function is the solution if the equation

$$Du = \sum_{i=1}^{n} \sum_{k=1}^{n} \frac{\partial u_k}{\partial x_i} e_i e_k = 0,$$

where $D = \sum_{i=1}^{n} \frac{\partial}{\partial x_i}$ denotes the so-called Dirac operator and $u = \sum_{k=1}^{n} u_k e_k$ is a Clifford-valued (vector) function. The classical Dirac operator D in $\mathbb{R}^n$ decomposes as $D = |D|\mathcal{H} = \mathcal{F}^{-1}\left(|\underline{\omega}|\frac{-i\underline{\omega}}{|\underline{\omega}|}\right)$, where $\mathcal{H} = \mathcal{F}^{-1}\left(\frac{-i\underline{\omega}}{|\underline{\omega}|}\right)$ is the Riesz-Hilbert transform and $\mathcal{F}^{-1}$ the inverse Fourier transform. The Hilbert-Riesz transform is a singular integral integral operator, i.e. a Fourier multiplier in $L^p, 1 < p < \infty$, and the Fourier transform is a homogeneous polynomial of degree zero.

We consider a generalized Hilbert-Riesz transforms and quasi-monogenic functions related to a generalized Dirac operator D_m defined in Fourier domain by $\mathcal{F}(D_m) = |\xi| \sum_{i=1}^{n} m_i(\xi)$. The generalized Riesz transforms build the generalized Riesz-Hilbert transform $\mathcal{H} = \sum_{j=1}^{n} R_j e_j$, where $R_j f = \mathcal{F}^{-1}(m_j(\underline{\omega})\hat{f}(\underline{\omega})$, i.e. a Fourier multiplier with Fourier symbol m_j. We use Hörmander-Mikhlin Multiplier theorem [2] to prove that all R_j are Fourier multipliers and singular integral operators.

The main result presented in our talk is concerned with boundary values of quasi-monogenic functions and the space $H^1(\mathbb{R}^n)$. Suppose $F = f + \sum_{j=1}^{n}(R_j f)e_j \in L^p(\mathbb{R}^n)$ and $G = g + \sum_{j=1}^{n}(R_j g)e_j \in L^q(\mathbb{R}^n)$ are boundary values of monogenic functions then the product FG can be written as

$$FG = fg - \sum_{j=1}^{n}(R_j f)(R_j g) + \sum_{j=1}^{n}(f R_j g + g R_j f)e_j + \sum_{j<k}(R_j f R_k g - R_j g R_k f)e_{jk}.$$

We will prove that the three sets

(i) $I_0 = \{fg - (\mathcal{H}f)(\mathcal{H}g), f \in L^p(\mathbb{R}^n,\ g \in L^q(\mathbb{R}^n)\}$,
(ii) $I_1 = \{f(R_j g) + g(R_j f), f \in L^p(\mathbb{R}^n,\ g \in L^q(\mathbb{R}^n)\}, 1 \leq j \leq n\}$,
(iii) $I_2 = \{(R_j f)(R_k g) - (R_j g)(R_k f), f \in L^p(\mathbb{R}^n,\ g \in L^q(\mathbb{R}^n)\}, 1 \leq j < k \leq n\}$.

span the space $H^1(\mathbb{R}^n)$. The main result in [1] proves it for $n = 1$, in [4] the result is extended to all n and the classical Riesz transforms. The prove is based on [3].

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Carathéodory-Fejér type interpolation problems for Stieltjes-class functions

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Let $\mathcal{P}$ denote the Pick class of analytic self-mappings of the upper half plane. The Stieltjes class $\mathcal{S}$ can be defined as the class of Pick functions $f(z)$ such that $zf(z)$ is also in $\mathcal{P}$. Stieltjes functions turn out to be analytic on $(-\infty, 0)$ and their restrictions to $(-\infty, 0)$ are characterized as nonnegative operator monotone functions on $(-\infty, 0)$. The Carathéodory-Fejér problem $\mathcal{CF}_n(\rfloor S, x_0)$ consists of finding a function $f \in \mathcal{S}$ with prescribed $f^{(j)}(x_0) = c_j$ $(j = 0, \ldots, n-1)$ if $x_0 \in \mathbb{C} \backslash (-\infty, 0)$. If $x_0 \geq 0$, then by $f^{(j)}(x_0)$ we mean the limit of $f^{(j)}(z)$ as z tends to x_0 nontangentially.

Roughly speaking, the problem $\mathcal{CF}_n(\mathcal{S}, x_0)$ has a solution if and only if two *Pick matrices* constructed from interpolation data are positive semidefinite, and the problem is indeterminate if and only if both matrices are positive definite, in which case the solution set is parametrized by a linear fractional formula with a free Stieltjes-class parameter. However, this is exactly the case if either $x_0 \notin \mathbb{R}$ or $x_0 < 0$ and n is even. It is almost the case if $x_0 = 0$ or $x_0 > 0$, n is even and all target values are real. In the talk, we will focus on the two remaining cases: (1) $x_0 < 0$ and n is odd and (2) $x_0 > 0$ and the target values are not necessarily real.

Lie algebraic solution of the Schrödinger equation

Roman Buniy
Chapman University

We give explicit solutions of the Schrödinger equation for the Hamiltonians which are linear combinations of generators of low-dimensional Lie algebras. As a simple application of these results, we solve the (surprisingly nontrivial) problem of a quantum-mechanical free particle of varying mass in a one-dimensional infinite potential well with moving boundaries.

Operator theory for discrete Clifford analysis

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Due to a fast growing of computational power one observes an increasing interest in discrete structures as counterparts of existent continuous structures. These discrete structures have applications in several practical problems and this is particularly visible in the finite element method, which nowadays relays on a finite element discretization modelated directly on the mesh. While the two dimensional case is well understood and has applications to problems in probability and statistical physics (see D. Chelkak, S. Smirnov, to name a few) the higher dimensional case is only recently being developed. Here, we should mention the approach pioneered by K. Gürlebeck and W. Sprößig in the end of the eighties, based on potential theory. Unfortunately, this approach has a drawback of having no overall operator theory, namely, no discrete equivalent notions to elliptic operators, strongly singular operators, and so on. In this talk we present the germ of a discrete operator theory in Clifford analysis based on a discrete pseudo-differential operator calculus with emphasis on its application to the potential-theoretical approach to discrete Clifford analysis.

The S-spectrum approach to fractional diffusion processes

Fabrizio Colombo
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In this talk we show an application of the spectral theory based on the notion of S-spectrum to fractional diffusion process. Precisely, we consider the Fourier law for the propagation of the heat in non homogeneous materials, that is the heat flow is given by the vector operator:

$$T = e_1\, a(x)\partial_{x_1} + e_2\, b(x)\partial_{x_2} + e_3\, c(x)\partial_{x_3}$$

where e_ℓ, $e_\ell = 1, 2, 3$ are orthogonal unit vectors in $\mathbb{R}^3$, a, b, c are given real valued functions that depend on the space variables $x = (x_1, x_2, x_3)$, and possibly also on time. Using the H^∞-version of the S-functional calculus we define fractional powers of quaternionic operators, which contain, as a particular case, the vector operator T. Hence, we can define the non-local version T^α, for $\alpha \in (0, 1)$, of the Fourier law defined by T. We will see how we have to interpret T^α, when we introduce the so called: "The S-spectrum approach to fractional diffusion processes". This new method allows us to enlarge the class of fractional diffusion and fractional evolution problems that can be defined and studied using the spectral theory based on the S-spectrum for vector operators.

Inverse problem for generalized de Branges matrices

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de Branges-Pontryagin spaces $B(E)$ with negative index k of entire vector valued functions based on an entire matrix valued function $E(z)$ (called the de Branges matrix) are studied. A characterization of those spaces $B(E)$ that are invariant under the generalized backward shift operator that extends known results when k=0 is given. The theory of rigged de Branges-Pontryagin spaces is developed and then applied to obtain an embedding of de Branges matrices with negative squares in generalized J-inner matrices. A formula for factoring an arbitrary generalized J-inner entire matrix valued function into the product of a singular factor and a perfect one is found analogous to the known factorization formulas for J-inner matrix valued functions.

On slice polyanalytic functions of a quaternionic variable

Kamal Diki
Politecnico di Milano, Italy

In this talk, we consider slice polyanalytic functions of a quaternionic variable and prove some of their properties. Then, the results obtained are used to study the counterparts of the Bergman and Fock spaces in this new setting. In particular, we give explicit expressions of their reproducing kernels (Joint work with Daniel Alpay and Irene Sabadini).

Dual-symmetric Electromagnetism, a Clifford-algebraic approach

Justin Dressel
Chapman University

We show how to use Minkowski spacetime Clifford algebra to efficiently describe electromagnetism. The electric and magnetic fields are combined into a single complex and frame-independent bivector field, which generalizes the Riemann-Silberstein complex vector that has recently resurfaced in studies of the single photon wavefunction. The complex structure of spacetime also underpins the emergence of electromagnetic waves, circular polarizations, the normal variables for canonical quantization, the distinction between electric and magnetic charge, complex spinor representations of Lorentz transformations, and the dual (electric-magnetic field exchange) symmetry that produces helicity conservation in vacuum fields. This latter symmetry manifests as an arbitrary global phase of the complex field, motivating the use of a complex vector potential, along with an associated transverse and gauge-invariant bivector potential, as well as complex (bivector and scalar) Hertz potentials.

Wasserstein geometry and applications

Tryphon T. Georgiou
University of California, Irvine

We will discuss recent advances in the theory of Monge-Kantorovich optimal mass transport, including second order calculus and non-commutative transport, and detail key results and applications that have motivated the development of the field.

News from Publishing

Thomas Hempfling
Birkhauser Basel

We focus on recent developments in Publishing in general and on activites within Springer-Nature in particular. Some topics are MOOCs, Sci-Graph, compact deals, and author services, to name a view.

W-Markov measures, transfer operators, wavelets and multiresolutions

Palle Jorgensen
University of Iowa

In a general setting we solve the following inverse problem: Given a positive operator R, acting on measurable functions on a fixed measure space $(X, \mathcal{B}_X)$, we construct an associated Markov chain. Specifically, starting with a choice of R (the transfer operator), and a probability measure μ_0 on $(X, \mathcal{B}_X)$, we then build an associated Markov chain $T_0, T_1, T_2, \ldots$, with these random variables (r.v) realized in a suitable probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and each r.v. taking values in X, and with T_0 having the probability μ_0 as law. We further show how spectral data for R, e.g., the presence of R-harmonic functions, propagate to the Markov chain. Conversely, in a general setting, we show that every Markov chain is determined by its transfer operator. In a range of examples we put this correspondence into practical terms: (i) iterated function systems (IFS), (ii) wavelet multiresolution constructions, and (iii) IFSs with random control. Our setting for IFSs is general as well: a fixed measure space $(X, \mathcal{B}_X)$ and a system of mappings τ_i, each acting in $(X, \mathcal{B}_X)$, and each assigned a probability, say p_i which may or may not be a function of x. For standard IFSs, the p_i's are constant, but for wavelet constructions, we have functions $p_i(x)$ reflecting the multi-band filters which make up the wavelet algorithm at hand. The sets $\tau_i(X)$ partition X, but they may have overlap, or not. For IFSs with random control, we show how the setting of transfer operators translates into explicit Markov moves: Starting with a point $x \in X$, the Markov move to the next point is in two steps, combined yielding the move from $T_0 = x$ to $T_1 = y$, and more generally from T_n to T_{n+1}. The initial point x will first move to one of the sets $\tau_i(X)$ with probability p_i, and once there, it will choose a definite position y (within $\tau_i(X)$), now governed by a fixed law (a given probability distribution). For Markov chains, the law is the same in each move from T_n to T_{n+1}.

Joint work with Daniel Alpay and Izchak Lewkowicz.

The power of being positive (semi-definite)

Amir Kalev
University of Maryland

Quantum tomography the generic protocol for characterizing quantum states and processes. However, since quantum states and processes are characterized by exponential number of parameters (in terms of the number of subsystems), to characterize quantum systems we must design more efficient tomographic protocols. In this talk I will describe how we may take advantage of prior information about the state of the system to improve data acquisition schemes of quantum state tomography. In particular, I will show how the statistical property of quantum states, i.e., that they are represented by a normalized positive semi-definite matrix, allow us to design more manageable tomographic schemes, such as compressed sensing quantum tomography.

Singular Schrödinger operators with predefined spectrum

Andrii Khrabustovskyi
Technische Universität Graz, Austria

We construct a singular Schrödinger operator on a compact interval with predefined essential spectrum and predefined finite part of the discrete spectrum. The required structure of the spectrum is realized by a special choice of a sequence of point interactions of δ and/or δ' types.
Our construction is inspired by a celebrated paper [1] and its sequel [2], where similar problem was treated for Neumann Laplacians on bounded domains.
This is a joint work with Jussi Behrndt (TU Graz).

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On the indefinite multidimensional truncated moment problem

David Kimsey
Newcastle University

Given a truncated multisequence s and non-negative integers $\kappa_{\pm}$, we exhibit necessary and sufficient conditions for s to have a representing measure $\mu = \mu_+ - \mu_-$, where $\operatorname{card}\operatorname{supp}\mu_{\pm} = \kappa_{\pm}$. We shall see that necessary and sufficient conditions can be formulated in terms of a rank preserving Hankel extension asuch that the polynomial ideal which models the aforementioned linear dependence is real radical. In the case that $\kappa_- = 0$, then our main result collapses to Curto and Fialkow's well-known flat extension theorem. The proof of the main result relies on some theory of Pontryagin spaces and also some basic results in commutative algebra.

Inverse source problems and approximation, with applications

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From joint works with L. Baratchart, S. Chevillard, M. Clerc, J.-P. Marmorat, K. Mavreas, C. Papageorgakis, T. Papadopoulo.

We consider the following Poisson-Laplace elliptic partial differential equation in $\mathbb{R}^3$ with source term in divergence form:

$$\Delta u = \operatorname{div} J = \sum_{q=1}^{Q} p_q \cdot \nabla \delta_{C_q} , \quad J = \sum_{q=1}^{Q} p_q \, \delta_{C_q} ,$$

for a $\mathbb{R}^3$ valued distribution J that consists in a linear combination of $Q \geq 1$ pointwise Dirac masses at locations C_q within a bounded domain $\Omega \subset \mathbb{R}^3$ with moments $p_q \in \mathbb{R}^3$. Inverse source problems aim at recovering the distribution J or some of its characteristics, thereby the quantity Q of pointwise sources, together with their locations C_q and moments p_q, from available partial (Dirichlet or Neumann or Cauchy) data of the associated solution u or (and) of components of its associated field ∇u. Although these are ill-posed problems, uniqueness of solutions may be granted under sufficient conditions (injectivity of the forward "source-to-data" map), though such issues remain unstable (lack of continuity of corresponding inverse "data-to-source" operators) and need to be regularized, which we do by setting them as best constrained approximation questions.
We will discuss situations where data are available on parametrized surfaces (spheres, cylinders) surrounding Ω, whence the support of J and the sources C_q, more specifically on series of circles contained in such surfaces (and surrounding Ω as well). This framework allows us to express the restriction to such a circle of the given data as that of a function of the complex variable that admits Q poles (and branched singularities) inside the corresponding disk. Estimating these singularities together with there quantity Q is the first step that we perform using best uniform (Adamjan-Arov-Krein) or quadratic meromorphic / rational approximation techniques in the complex plane. Those furnish a way to regularize the issue, by constraining the degree. Once this is done in every circles where data are available, the obtained 2-dimensional singularities are clustered together in order to furnish an estimation of the 3-dimensional sources C_q. Indeed, the locations of the singularities can be explicitely related to those of the sources, using the circles parametrization. Finally, the computation of the moments p_q is a linear problem.
Moreover, such a circular setup is suitable in view of practical physical applications in electromagnetism that we will describe. Indeed, from Maxwell's equations, the above elliptic PDE models the behaviour of electromagnetic potentials u under quasi-static assumptions. This is the situation in electroencephalography (EEG) and magnetoencephalography (MEG), imaging modalities for clinical and functional purposes in medical engineering and neurosciences. Their aim is to estimate a source term (the primary cerebral current J) supported within the brain, from pointwise measurements of the produced electrical potential u, taken by electrodes on the scalp in EEG (the normal current $\partial_n u$ vanishes outside the head), or of a component of the produced magnetic field taken by

magnetic sensors outside the head in MEG. In spherical head geometries, and after preliminary time-space separation and data transmission or extension steps, we solve the corresponding inverse source problems using the above approach, [2]. This is also the case of a magnetization problem arising in paleomagnetism, a branch of planetary sciences, where J stands for the magnetization contained in a piece of rock, that generates a tiny magnetic field ∇u of which the components can be measuremed using magnetometers. It happens that the magnetic field generated by the remanent magnetization yet contained in Moon rocks can be measured on a few circles lying on cylinders by a specific spinner magnetometer, giving quite sparse data, [1]. From those data, the aim is to estimate the moments p_q, under the assumption that there is $Q = 1$ or 2 dipolar sources within the rock. This problem is solved using the above process as well. For both issues, numerical results will be shown, from synthetic and actual provided by our partners, obtained with the software FindSources3D*.

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*See `www-sop.inria.fr/apics/FindSources3D/`

State-space realization of Tensor product or Composition of rational functions

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State space realization of product of matrix-valued rational functions, in terms of realizations of the original functions, is classical. We here extend this result to *(i) tensor-product* and to *(ii) composition* of rational functions. As time permits, various applications will be discussed.

Joint work with Daniel Alpay.

A Hybrid distribution for heavy tailed data modeling

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One of the main issues in the statistical literature of extremes concerns the tail index estimation, closely linked to the determination of a threshold above which a Generalized Pareto Distribution (GPD) can be fitted. Approaches to this estimation may be classified into two classes, one using standard Peak Over Threshold (POT) methods, in which the threshold to estimate the tail is chosen graphically according to the problem, the other suggesting self-calibrating methods, where the threshold is algorithmically determined. The approach of this talk belongs to this second class proposing a hybrid distribution for heavy tailed data modeling, which links a normal (or lognormal) distribution to a GPD via an exponential distribution that bridges the gap between mean and asymptotic behaviors. A unsupervised algorithm is then developed for estimating the parameters of this model. Some application to real data from neuroscience and finance are given.

This is joint work N. Debbabi and M. Kratz

Keywords: Extreme Value Theory; Gaussian distribution; Generalized Pareto Distribution; Heavy tailed data; Hybrid model

A solution of the Boussinesq equation using evolutionary vessels

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In this work, we present a solution of the Boussinesq equation, which models shallow water waves. A powerful inverse scattering approach of Deift-Tomei-Trubowitz was published in 1982 for solving this equation in the Schwartz class. We rewrite the corresponding Lax pair in a matrix form and show that a recently developed theory of evolutionary nodes is applicable. As a result, we can locally solve this equation with analytic initial conditions on the line, providing a formula for the tau function, which defines where solutions exist globally.

Some Electromagnetic Wave Propagation Models for Complex Media: An Operator Theoretical Perspective

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The study of Maxwell's equations in complex media (metamaterials) has come to a considerable attention within the last fifteen to twenty years. Such problems have been extensively studied mostly in the time-harmonic case. In this paper we focus on the time-dependent case. Following the ideas of [1], [2, Chapter 6] such problems can be embedded into a general class of operator equations with a unified solution theory. We discuss various particular models with complex material laws associated with the classical Maxwell equations. Well-posedness results for a large class of media can be obtained.

References

[1] R. Picard. A Structural Observation for Linear Material Laws in Classical Mathematical Physics. *Math. Methods Appl. Sci.*, 32(14):1768–1803, 2009.
[2] R. Picard and D. F. McGhee. *Partial Differential Equations: A unified Hilbert Space Approach*, volume 55 of *De Gruyter Expositions in Mathematics*. De Gruyter. Berlin, New York, 2011.

Vessel theory over Pontryagin spaces with applications to system theory

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In this talk we extend the vessel theory, or equivalently, the theory of overdetermined $2D$ systems to the Pontryagin space setting. The associated transfer function becomes a mapping with a finite number of negative squares between certain vector bundles defined on a compact Riemann Surface. Special focus is given to the realization theorems for the various characteristic functions.

The Pontryagin vessel theory allows us to develop and study an indefinite version of the de Branges Rovnyak spaces over real compact Riemann surfaces, i.e. reproducing kernel Pontryagin spaces of analytic sections defined on real compact Riemann surfaces. Furthermore, we utilize the indefinite de Branges-Rovnyak theory on compact Riemann surfaces in order to present a Beurling type theorem on indefinite Hardy spaces on finite bordered Riemann surfaces.

These results are based on a joint work with Daniel Alpay and Victor Vinnikov.

Evaluations of non-commutative rational functions on stably finite algebras

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The theory of non-commutative (nc) rational functions which are regular at 0 is well known and studied, in terms of their minimal realizations: any such function admits a unique minimal realization centered at 0 and the domain of the function coincides with the invertibility set of the (resolvent of the) realization.

In this talk we present a generalization of these ideas to the case where the centre is non-scalar; We show the existence and uniqueness of a minimal realization for any nc rational function, prove that every point in the domain of the function belongs (in some sense) to the domain of the realization and that one can evaluate the function by evaluating the realization.

As a corollary we get a new proof that the domain of a nc rational function coincides with the invertibility set of its minimal realization, in the case where the centre is 0. This is a joint work with Victor Vinnikov.

On some Radon-type transforms in hypercomplex analysis

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In this talk we discuss some Radon-type transforms in higher dimensions, in the framework of monogenic functions. We first define a version of the Radon transform based on Szegő kernels. We shall show how the transform is explicitely computed on axially monogenic functions of degree k. We also introduce a Szegő-Radon projection which may be abstractly defined as the orthogonal projection of a suitable Hilbert module of square integrable left monogenic functions onto the closed submodule of monogenic functions spanned by the monogenic plane waves. We then discuss a Bargmann-Radon transform.This transform can be expressed in integral form using a suitable kernel.

If time permits, we shall introduce a version of the Szegő-Radon transform for a class of holomorphic functions in the unit Lie ball in $\mathbb{C}^m$.

Propagators of the Schrödinger equation: Time evolution of superoscillations

Peter Schlosser
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Superoscillation is the phenomenon that functions can (locally) oscillate faster than any of their Fourier components. This strange and paradoxical behaviour was first found by Y. Aharonov and M. Berry who constructed and investigated specific functions having this property. Since the origin of such functions is physics, with the motivation e.g. to get higher resolutions of optical measurements, we continue the mathematical investigation of the theory of superoscillations. Our work now concentrates on superoscillating functions as initial condition of the time dependent Schrodinger equation. Therefore, we derive propagators for specific potentials and treat them as operators between function spaces which are suitable to describe superoscillations. The problem of the persistence of superoscillations over time has been studied intensively in the last decade. The main goal is to continue the investigation of the persistence of superoscillations over time for the case of the Dirac delta potential.

On the Hilbert formulas on the unit circle for α-hyperholomorphic function theory

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Let us denote by $\mathbb{S}$ the unit circle in the complex plane $\mathbb{C}$ and given a limit function f $\mathbb{S}$. set $g(\theta) := f(e^{i\theta})$, $0 \le \theta < 2\pi$, and $g = g_1 + ig_2$. Then the real components g_1 and g_2 of g are related by the following formulas known as the Hilbert formulas for the unit disc:

$$\mathcal{M}[g_1] + \mathcal{H}[g_2] = g_1,$$
$$\mathcal{M}[g_2] - \mathcal{H}[g_1] = g_2,$$

where M and $\mathcal{H}$ are given by

$$\mathcal{H}[g](\theta) := \frac{1}{2\pi}\int_0^{2\pi} \cot\frac{\tau-\theta}{2} g(\tau)d\tau, \quad \theta \in [0, 2\pi), \tag{0.4}$$
$$M[g] := \frac{1}{2\pi}\int_0^{2\pi} g(\tau)d\tau,$$

which are both defined on the linear space of real valued Hölder continuous functions $C^{0,\mu}(\mathbb{S}, \mathbb{R})$, $\mu \in (0, 1]$.
The integral $\mathcal{H}[g]$ (well-defined on $C^{0,\mu}(\mathbb{S}, \mathbb{R})$, $\mu \in (0, 1]$) is understood in the sense of the Cauchy principal value, generating the so-called Hilbert operator with (real) kernel $\frac{1}{2\pi}\cot\frac{\tau-\theta}{2}$.
The Hilbert operator (0.4) is well-known transformation in mathematics and in signal processing; for example, in geophysics and astrophysics it deals with input signals. Examples of this type of signals are seismic, satellite and gravitational data; and the Hilbert operator proves to be useful for a local analysis of them, providing a set of rotation-invariant local properties: the local amplitude, local orientation and local phase.
Various analogues of the Hilbert formulas on the unit sphere keep interest until our days. In this talk we give some analogues of the Hilbert formulas on the unit circle for α–hyperholomorphic function theory when α is an arbitrary complex quaternion number.

Talk based on joint works with J. Bory Reyes, R. Abreu Blaya, M. A. Pérez-de la Rosa.

On a certain trigonometric series

Ahmed Sebbar
Chapman University

A famous theorem of Arne Beurling asserts that the Riemann hypothesis holds if and only if the characteristic function $\chi_{[0,1]}$ can be approximated by linear combinations of e_α in $L^2(0,1)$, where

$$e_\alpha(x) = [\frac{\alpha}{x}] - \alpha[\frac{1}{x}].$$

In connexion with this theorem we study the very interesting trigonometric series, called Flett series or Hardy-Littlewood series

$$\sum_{n\geq 1} \frac{1}{n} \sin(\frac{x}{n}).$$

Superoscillations and continuous endomorphisms on spaces of entire functions

Daniele Struppa
Chapman University

The study of superoscillations, and especially the question of their longevity when evolved according to the Schrodinger equation, leads naturally to the study of the continuity of large class of operators on spaces of entire functions. In this talk I will show the connection between these two problems, and I will present a few recent results that originated in this context. The results I will describe arise from joint works with Aoki, Colombo, Ishimura, Okada, Sabadini, and Uchida.

References:

Aoki T., Colombo F., Sabadini I., Struppa D.C., Continuity theorems for a class of convolution operators and applications to superoscillations, Ann. Mat. Pura e Applicata, 2018.

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Well-posedness for a general class of differential inclusions

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We consider an abstract class of differential inclusions of the form

$$(u, f) \in \partial_t \mathcal{M} + \mathcal{N} + \mathcal{A},$$

where ∂_t denotes the temporal derivative, $\mathcal{A}$ is a maximal monotone relation on some Hilbert space H and $\mathcal{M}, \mathcal{N}$ are bounded linear operators acting in space-time. This class of problems covers a lot of problems from mathematical physics including non-autonomous non-linear problems with delay and hysteresis. We will prove the well-posedness of the problem above in a pure Hilbert space setting under weak assumptions on the operators involved and illustrate its applicability by several examples from mathematical physics.

References

[1] S. Trostorff. Well-posedness for a general class of differential inclusions. Technical report, TU Dresden, 2018. arXiv: 1808.00224, submitted.

Operator based approach to $\mathcal{PT}$ symmetric problems on wedge shaped contours

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Since 20 years specific Sturm-Liouville equations with polynomial potentials on non-real contours are studied within non-Hermitian quantum mechanics. In theoretical physics this is often called

$\mathcal{PT}$ Symmetric Quantum Mechanics.

Here $\mathcal{PT}$ symmetry indicates that the corresponding Hamiltonian commutes with the joint action of $\mathcal{P}$ and $\mathcal{T}$, where $\mathcal{P}$ stands for parity and $\mathcal{T}$ for time reversal.
The underlying idea in $\mathcal{PT}$ symmetric quantum mechanics is to assume that the Hamiltonain is only $\mathcal{PT}$ symmetric, contrary to classical quantum mechanics where it is selfadjoint. Nevertheless, there is the hope in theoretical physics that $\mathcal{PT}$ symmetric Hamiltonians possesses real spectrum.

We will give an introduction to $\mathcal{PT}$ symmetric quantum mechanics and present an operator theoretic approach: The right spaces, domains, symmetries, and boundary conditions. After a mathematically rigid formulation of the spectral problem in terms of operators we are able to discuss the location of the (point) spectrum and we determine areas in the complex plane which are free of eigenvalues.

This is based on joint works with R. Hryniv (Lviv and Rzeszów) and F. Leben (Ilmenau).

Script Geometry

Adrian Vajiac
Chapman University

In this talk we describe the foundation of a new kind of discrete geometry and calculus called Script Geometry. It allows to work with more general meshes than classic simplicial complexes. We provide the basic definitions as well as several examples, like the Klein bottle and the projective plane. Furthermore, we also introduce the corresponding Dirac and Laplace operators which should lay the groundwork for the development of the corresponding discrete function theory. This a joint collaborative work with P. Cerejeiras, U. Kähler, F. Sommen and M.B. Vajiac.

Interpolation problems for discrete analytic functions

Dan Volok
Kansas State University

Compared to the continuous case, interpolation of discrete analytic function seems more difficult for two reasons: 1. the usual point-wise product of functions is incompatible with discrete analyticity, and 2. in view of discrete Cauchy-Riemann equations, for some choices of interpolation points the corresponding function values cannot be prescribed arbitrarily. In this talk we describe those sets of points on the integer lattice in the complex plane which are suitable for discrete analytic interpolation and the solution of the related basic interpolation problem.

This is a joint work with D. Alpay

Generalizations of Quaternions Based on the Physics of N qubits

Mordecai Waegell
Chapman University

I will discuss an interesting connection between the space of N-qubit quantum states, and generalized quaternions. For a single qubit, the mapping between a quantum state and a standard quaternion is quite straightforward, since the quaternion components i, j, k, and 1 can be trivially mapped to the 2×2 Pauli matrices in Hilbert space, and these give a representation for any single-qubit state. This mapping can be generalized by defining a new set of 16 generalized quaternion components which are mapped to the 16 tensor products of the single-qubit Pauli matrices and identity. This set contains a much richer structure, with some of the generalized quaternion components now commuting with one another. Furthermore, the phenomenon of quantum entanglement, which relates to the factorizability of qauntum states into separate vector spaces for two or more systems, can now be directly related to the commutation structure of a generalized quaternion. There may also be interesting connections to the phenomenon of quantum contextuality, which will depend on how quantum measurements are mapped into the space of the generalized quaternions. Furthermore, we can extend this generalization to any N, resulting in generalized quaternions with 4^N components. These, and other interesting aspects of this special class of numbers will be examined.

Nonlocal homogenisation problems see the boundary conditions

Marcus Waurick
University of Strathclyde

In standard homogenisation problems for divergence-form problems with multiplication operators as coefficients, it can be shown that the limit is independent of the boundary conditions attached. In particular, the effective behaviours for Dirichlet boundary values problems and Neumann boundary value problems coincide. Recalling the concept of nonlocal H-convergence, we shall show that this is no longer the case for nonlocal operators as coefficients. In the talk, I will explain and describe the above terminology and show the lack of independence of the boundary conditions by means of an example.
The results can be found in `https://arxiv.org/pdf/1804.02026.pdf`

About Cauchy-Weil formula and Bergman-Weil f-adic expansions

Alain Yger
IMB, Université de Bordeaux, Talence, France

I will recall a classical integral representation formula due to André Weil (1933), with some of its consequences, and relate it in particular to division-interpolation problems in Hardy spaces. The results I will present arise from some joint work with Alekos Vidras (Nicosia), Martin Sombra (Barcelona) and work in progress with Daniel Alpay.